

STEP III, 1999, Q6

- 6 A closed curve is given by the equation

$$x^{2/n} + y^{2/n} = a^{2/n} \quad (*)$$

where n is an odd integer and a is a positive constant. Find a parametrization $x = x(t)$, $y = y(t)$ which describes the curve anticlockwise as t ranges from 0 to 2π .

Sketch the curve in the case $n = 3$, justifying the main features of your sketch.

The area A enclosed by such a curve is given by the formula

$$A = \frac{1}{2} \int_0^{2\pi} \left[x(t) \frac{dy(t)}{dt} - y(t) \frac{dx(t)}{dt} \right] dt.$$

Use this result to find the area enclosed by (*) for $n = 3$.



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