



STEP III, 1999, Q5

- 5 The sequence u_0, u_1, u_2, \dots is defined by

$$u_0 = 1, \quad u_1 = 1, \quad u_{n+1} = u_n + u_{n-1} \quad \text{for } n \geq 1.$$

Prove that

$$u_{n+2}^2 + u_{n-1}^2 = 2(u_{n+1}^2 + u_n^2).$$

Using induction, or otherwise, prove the following result:

$$u_{2n} = u_n^2 + u_{n-1}^2 \quad \text{and} \quad u_{2n+1} = u_{n+1}^2 - u_{n-1}^2$$

for any positive integer n .



NextStepMaths.com

To view mark schemes, fully worked solutions and examiner's comments, and for more details about tutoring and other services offered, go to

NextStepMaths.com