

## STEP III, 1999, Q2

- 2 (i) Let  $f(x) = (1 + x^2)e^x$ . Show that  $f'(x) \geq 0$  and sketch the graph of  $f(x)$ . Hence, or otherwise, show that the equation

$$(1 + x^2)e^x = k,$$

where  $k$  is a constant, has exactly one real root if  $k > 0$  and no real roots if  $k \leq 0$ .

- (ii) Determine the number of real roots of the equation

$$(e^x - 1) - k \tan^{-1} x = 0$$

in the cases (a)  $0 < k \leq 2/\pi$  and (b)  $2/\pi < k < 1$ .



# NextStepMaths.com

To view mark schemes, fully worked solutions and examiner's comments, and for more details about tutoring and other services offered, go to

[NextStepMaths.com](http://NextStepMaths.com)