

## STEP III, 1999, Q14

- 14 In the basic version of Horizons (H1) the player has a maximum of  $n$  turns, where  $n \geq 1$ . At each turn, she has a probability  $p$  of success, where  $0 < p < 1$ . If her first success is at the  $r$ th turn, where  $1 \leq r \leq n$ , she collects  $r$  pounds and then withdraws from the game. Otherwise, her winnings are nil. Show that in H1, her expected winnings are

$$p^{-1} [1 + nq^{n+1} - (n+1)q^n] \quad \text{pounds,}$$

where  $q = 1 - p$ .

The rules of H2 are the same as those of H1, except that  $n$  is randomly selected from a Poisson distribution with parameter  $\lambda$ . If  $n = 0$  her winnings are nil. Otherwise she plays H1 with the selected  $n$ . Show that in H2, her expected winnings are

$$\frac{1}{p} (1 - e^{-\lambda p}) - \lambda q e^{-\lambda p} \quad \text{pounds.}$$



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