STEP III, 1999, Q12

12 In the game of endless cricket the scores X and Y of the two sides are such that

$$\mathrm{P}(X = j,\ Y = k) = e^{-1}\frac{(j+k)\lambda^{j+k}}{j!k!},$$

for some positive constant λ, where $j, k = 0,\ 1,\ 2,\ \ldots$.

(i) Find $\mathrm{P}(X + Y = n)$ for each $n > 0$.

(ii) Show that $2\lambda e^{2\lambda - 1} = 1$.

(iii) Show that $2xe^{2x-1}$ is an increasing function of x for $x > 0$ and deduce that the equation in (ii) has at most one solution and hence determine λ.

(iv) Calculate the expectation $\mathrm{E}(2^{X+Y})$.