

## **STEP III, 1999, Q1**

1 Consider the cubic equation

$$x^3 - px^2 + qx - r = 0,$$

where  $p \neq 0$  and  $r \neq 0$ .

- (i) If the three roots can be written in the form  $ak^{-1}$ ,  $a$  and  $ak$  for some constants  $a$  and  $k$ , show that one root is  $q/p$  and that  $q^3 - rp^3 = 0$ .
- (ii) If  $r = q^3/p^3$ , show that  $q/p$  is a root and that the product of the other two roots is  $(q/p)^2$ . Deduce that the roots are in geometric progression.
- (iii) Find a necessary and sufficient condition involving  $p$ ,  $q$  and  $r$  for the roots to be in arithmetic progression.



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