

## STEP III, 1998, Q8

- 8 (i) Show that the line  $\mathbf{r} = \mathbf{b} + \lambda\mathbf{m}$ , where  $\mathbf{m}$  is a unit vector, intersects the sphere  $\mathbf{r} \cdot \mathbf{r} = a^2$  at two points if

$$a^2 > \mathbf{b} \cdot \mathbf{b} - (\mathbf{b} \cdot \mathbf{m})^2.$$

Write down the corresponding condition for there to be precisely one point of intersection. If this point has position vector  $\mathbf{p}$ , show that  $\mathbf{m} \cdot \mathbf{p} = 0$ .

- (ii) Now consider a second sphere of radius  $a$  and a plane perpendicular to a unit vector  $\mathbf{n}$ . The centre of the sphere has position vector  $\mathbf{d}$  and the minimum distance from the origin to the plane is  $l$ . What is the condition for the plane to be tangential to this second sphere?
- (iii) Show that the first and second spheres intersect at right angles (*i.e.* the two radii to each point of intersection are perpendicular) if

$$\mathbf{d} \cdot \mathbf{d} = 2a^2.$$



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