

STEP III, 1998, Q7

- 7 Sketch the graph of $f(s) = e^s(s - 3) + 3$ for $0 \leq s < \infty$. Taking $e \approx 2.7$, find the smallest positive integer, m , such that $f(m) > 0$.

Now let

$$b(x) = \frac{x^3}{e^{x/T} - 1}$$

where T is a positive constant. Show that $b(x)$ has a single turning point in $0 < x < \infty$. By considering the behaviour for small x and for large x , sketch $b(x)$ for $0 \leq x < \infty$.

Let

$$\int_0^{\infty} b(x) dx = B,$$

which may be assumed to be finite. Show that $B = KT^n$ where K is a constant, and n is an integer which you should determine.

Given that $B \approx 2 \int_0^{Tm} b(x) dx$, use your graph of $b(x)$ to find a rough estimate for K .



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