STEP III, 1998, Q2

2 Let

$$\mathrm{I}(a,b) = \int_0^1 t^a(1-t)^b \,\mathrm{d}t \qquad (a \geqslant 0,\ b \geqslant 0).$$

(i) Show that $\mathrm{I}(a,b) = \mathrm{I}(b,a)$,

(ii) Show that $\mathrm{I}(a,b) = \mathrm{I}(a+1,b) + \mathrm{I}(a,b+1)$.

(iii) Show that $(a+1)\mathrm{I}(a,b) = b\mathrm{I}(a+1,b-1)$ when a and b are positive and hence calculate $\mathrm{I}(a,b)$ when a and b are positive integers.