

## STEP III, 1998, Q11

- 11 Consider a simple pendulum of length  $l$  and angular displacement  $\theta$ , which is **not** assumed to be small. Show that

$$\frac{1}{2}l \left( \frac{d\theta}{dt} \right)^2 = g(\cos \theta - \cos \gamma),$$

where  $\gamma$  is the maximum value of  $\theta$ . Show also that the period  $P$  is given by

$$P = 2\sqrt{\frac{l}{g}} \int_0^\gamma (\sin^2(\gamma/2) - \sin^2(\theta/2))^{-\frac{1}{2}} d\theta.$$

By using the substitution  $\sin(\theta/2) = \sin(\gamma/2) \sin \phi$ , and then finding an approximate expression for the integrand using the binomial expansion, show that for small values of  $\gamma$  the period is approximately

$$2\pi\sqrt{\frac{l}{g}} \left( 1 + \frac{\gamma^2}{16} \right).$$



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