STEP II, 2024, Q6 MS

Question	Answer	Mark
6 i	For $n = 0$: $$T_0 = \frac{1}{2^0}\binom{0}{0} = 1$$	B1
	Assume that the result is true for $n = k$: $T_k = \frac{1}{2^{2k}}\binom{2k}{k}$	
	$T_{k+1} = \frac{2(k+1)-1}{2(k+1)} \cdot \frac{1}{2^{2k}}\binom{2k}{k}$	M1
	$T_{k+1} = \frac{1}{2^{2k}} \cdot \frac{2k+1}{2(k+1)} \cdot \frac{(2k)!}{(k!)^2}$ $T_{k+1} = \frac{1}{2^{2k}} \cdot \frac{2k+1}{2(k+1)} \cdot \frac{(2k)!}{(k!)^2} \cdot \frac{2k+2}{2(k+1)}$ $T_{k+1} = \frac{1}{2^{2(k+1)}} \cdot \frac{(2(k+1))!}{((k+1)!)^2}$ $T_k = \frac{1}{2^{2(k+1)}}\binom{2(k+1)}{k+1}$	M1 A1
	Hence, by induction: $T_n = \frac{1}{2^{2n}}\binom{2n}{n}$	A1
		[5]
ii	$a_r = \left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\cdots\left(-\frac{2r-1}{2}\right)\frac{(-1)^r}{r!}$	M1 A1
	$a_{r-1} = \left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\cdots\left(-\frac{2(r-1)-1}{2}\right)\frac{(-1)^{r-1}}{(r-1)!}$	M1
	Therefore, $a_r = a_{r-1} \cdot \left(-\frac{2r-1}{2}\right) \cdot \frac{-1}{r}$ $a_r = \frac{2r-1}{2r}a_{r-1}$	E1
	Since $a_0 = 1 = T_0$,	B1
	$a_r = T_r$ for $r = 0, 1, 2, \cdots$	A1
		[6]
iii	$$b_r = \frac{\left(\frac{3}{2} \cdot \frac{5}{2} \cdot \ldots \cdot \frac{(2r-1)}{2} \cdot \frac{(2r+1)}{2}\right)}{r!}$$	M1
	So, $\frac{b_r}{a_r} = 2r + 1$ Correctly argued for general terms	A1 E1
	$b_r = \frac{2r+1}{2^{2r}}\binom{2r}{r}$	A1
		[4]

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Question	Answer	Mark
6 iv	$(1-x)^{-1} = \Sigma_{r=0}^{\infty} x^r$	B1
	$(1 + x + x^2 + \cdots)(a_0 + a_1 x + a_2 x^2 + \cdots) = (b_0 + b_1 x + b_2 x^2 + \cdots)$	B1
	The term in x^n on the LHS is: $1 \cdot a_n x^n + x \cdot a_{n-1} x^{n-1} + \cdots + x^n \cdot a_0$	M1 A1
	Therefore, $b_n = \Sigma_{r=0}^{n} a_r$ as required.	A1
		[5]

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