

STEP II, 2024, Q5

- 5 (i) The functions f_1 and F_1 , each with domain \mathbb{Z} , are defined by

$$f_1(n) = n^2 + 6n + 11,$$

$$F_1(n) = n^2 + 2.$$

Show that F_1 has the same range as f_1 .

- (ii) The function g_1 , with domain \mathbb{Z} , is defined by

$$g_1(n) = n^2 - 2n + 5.$$

Show that the ranges of f_1 and g_1 have empty intersection.

- (iii) The functions f_2 and g_2 , each with domain \mathbb{Z} , are defined by

$$f_2(n) = n^2 - 2n - 6,$$

$$g_2(n) = n^2 - 4n + 2.$$

Find any integers that lie in the intersection of the ranges of the two functions.

- (iv) Show that $p^2 + pq + q^2 \geq 0$ for all real p and q .

The functions f_3 and g_3 , each with domain \mathbb{Z} , are defined by

$$f_3(n) = n^3 - 3n^2 + 7n,$$

$$g_3(n) = n^3 + 4n - 6.$$

Find any integers that lie in the intersection of the ranges of the two functions.



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