

STEP II, 2024, Q2 MS

Question	Answer	Mark
2 i	$(8 + x^3)^{-1} = \frac{1}{8} \left(1 + \frac{x^3}{8}\right)^{-1}$	
	$= \frac{1}{8} \left(1 - \frac{x^3}{8} + \frac{x^6}{64} - \frac{x^9}{512} + \dots\right)$	M1
	$= \frac{1}{8} \sum_{k=0}^{\infty} (-1)^k \left(\frac{x^3}{8}\right)^k$	A1
	$\int_0^1 \frac{x^m}{8 + x^3} dx = \int_0^1 \frac{1}{8} \sum_{k=0}^{\infty} \frac{(-1)^k}{2^{3k}} x^{m+3k} dx$	M1
	$= \frac{1}{8} \sum_{k=0}^{\infty} \left[\frac{(-1)^k}{2^{3k}} \frac{x^{m+3k+1}}{m+3k+1} \right]_0^1$	A1
	$= \sum_{k=0}^{\infty} \left(\frac{(-1)^k}{2^{3(k+1)}} \frac{1}{m+3k+1} \right)$	A1
		[5]
ii	$\sum_{k=0}^{\infty} \frac{(-1)^k}{2^{3(k+1)}} \left(\frac{1}{3k+3} \right) = \int_0^1 \frac{x^2}{8 + x^3} dx$	M1
	$\sum_{k=0}^{\infty} \frac{(-1)^k}{2^{3(k+1)}} \left(\frac{-2}{3k+2} \right) = \int_0^1 \frac{-2x}{8 + x^3} dx$	
	$\sum_{k=0}^{\infty} \frac{(-1)^k}{2^{3(k+1)}} \left(\frac{4}{3k+1} \right) = \int_0^1 \frac{4}{8 + x^3} dx$	
	$\sum_{k=0}^{\infty} \frac{(-1)^k}{2^{3(k+1)}} \left(\frac{1}{3k+3} - \frac{2}{3k+2} + \frac{4}{3k+1} \right)$	A1
	$= \int_0^1 \frac{x^2 - 2x + 4}{8 + x^3} dx$	
	$= \int_0^1 \frac{x^2 - 2x + 4}{(x+2)(x^2 - 2x + 4)} dx$	M1
	$= \int_0^1 \frac{1}{x+2} dx$	A1
$= [\ln(x+2)]_0^1 = \ln\left(\frac{3}{2}\right)$	B1	
	[5]	
iii	$\frac{72(2k+1)}{(3k+1)(3k+2)} = \frac{24}{3k+1} + \frac{24}{3k+2}$	M1
	$\sum_{k=0}^{\infty} \frac{(-1)^k}{2^{3(k+1)}} \frac{72(2k+1)}{(3k+1)(3k+2)} = \int_0^1 \frac{24x + 24}{8 + x^3} dx$	A1
	$= \int_0^1 \frac{2(x+8)}{x^2 - 2x + 4} - \frac{2}{x+2} dx$	M1
	$= \int_0^1 \frac{2(x-1)}{x^2 - 2x + 4} + \frac{18}{x^2 - 2x + 4} - \frac{2}{x+2} dx$	A1
	$= [\ln(x^2 - 2x + 4)]_0^1 \dots$	M1
	$\dots + \left[6\sqrt{3} \arctan\left(\frac{x-1}{\sqrt{3}}\right) \right]_0^1 \dots$	M1
	$\dots - [2\ln(x+2)]_0^1$	A1
	$= \ln 3 - \ln 4 - 2\ln 3 + 2\ln 2 + 6\sqrt{3} \cdot \frac{\pi}{6}$	A1
	$= \pi\sqrt{3} - \ln 3$	
		[10]



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