STEP II, 2024, Q2 MS

Question	Answer	Mark
2 i	$(8+x^3)^{-1}=\frac{1}{8}\left(1+\frac{x^3}{8}\right)^{-1}$	
	$=\frac{1}{8}\left(1-\frac{x^3}{8}+\frac{x^6}{64}-\frac{x^9}{512}+\cdots\right)$	M1
	$=\frac{1}{8}\sum_{k=0}^{\infty}(-1)^k\left(\frac{x}{2}\right)^{3k}$	A1
	$\int_0^1\frac{x^m}{8+x^3}dx=\int_0^1\frac{1}{8}\sum_{k=0}^{\infty}\frac{(-1)^k}{2^{3k}}x^{m+3k}\,dx$	M1
	$=\frac{1}{8}\sum_{k=0}^{\infty}\left[\frac{(-1)^k}{2^{3k}}\frac{x^{m+3k+1}}{m+3k+1}\right]_0^1$	A1
	$=\sum_{k=0}^{\infty}\left(\frac{(-1)^k}{2^{3(k+1)}}\frac{1}{m+3k+1}\right)$	A1
		[5]
ii	$\sum_{k=0}^{\infty}\frac{(-1)^k}{2^{3(k+1)}}\left(\frac{1}{3k+3}\right)=\int_0^1\frac{x^2}{8+x^3}dx$ $\sum_{k=0}^{\infty}\frac{(-1)^k}{2^{3(k+1)}}\left(\frac{-2}{3k+2}\right)=\int_0^1\frac{-2x}{8+x^3}dx$ $\sum_{k=0}^{\infty}\frac{(-1)^k}{2^{3(k+1)}}\left(\frac{4}{3k+1}\right)=\int_0^1\frac{4}{8+x^3}dx$	M1
	$\sum_{k=0}^{\infty}\frac{(-1)^k}{2^{3(k+1)}}\left(\frac{1}{3k+3}-\frac{2}{3k+2}+\frac{4}{3k+1}\right)$ $=\int_0^1\frac{x^2-2x+4}{8+x^3}dx$	A1
	$=\int_0^1\frac{x^2-2x+4}{(x+2)(x^2-2x+4)}dx$	M1
	$=\int_0^1\frac{1}{x+2}dx$	A1
	$=[\ln(x+2)]_0^1=\ln\left(\frac{3}{2}\right)$	B1
		[5]
iii	$\frac{72(2k+1)}{(3k+1)(3k+2)}=\frac{24}{3k+1}+\frac{24}{3k+2}$	M1 A1
	$\sum_{k=0}^{\infty}\frac{(-1)^k}{2^{3(k+1)}}\frac{72(2k+1)}{(3k+1)(3k+2)}=\int_0^1\frac{24x+24}{8+x^3}dx$	A1
	$=\int_0^1\frac{2(x+8)}{x^2-2x+4}-\frac{2}{x+2}dx$	M1 A1
	$=\int_0^1\frac{2(x-1)}{x^2-2x+4}+\frac{18}{x^2-2x+4}-\frac{2}{x+2}dx$	M1
	$=[\ln(x^2-2x+4)]_0^1\ldots$	M1
	$\ldots+\left[6\sqrt{3}\arctan\left(\frac{x-1}{\sqrt{3}}\right)\right]_0^1\ldots$	M1
	$\ldots-[2\ln(x+2)]_0^1$	A1
	$=\ln 3-\ln 4-2\ln 3+2\ln 2+6\sqrt{3}\cdot\frac{\pi}{6}$ $=\pi\sqrt{3}-\ln 3$	A1
		[10]