

STEP II, 2024, Q2

2 In this question, you need not consider issues of convergence.

(i) Find the binomial series expansion of $(8 + x^3)^{-1}$, valid for $|x| < 2$.

Hence show that, for each integer $m \geq 0$,

$$\int_0^1 \frac{x^m}{8 + x^3} dx = \sum_{k=0}^{\infty} \left(\frac{(-1)^k}{2^{3(k+1)}} \cdot \frac{1}{3k + m + 1} \right).$$

(ii) Show that

$$\sum_{k=0}^{\infty} \frac{(-1)^k}{2^{3(k+1)}} \left(\frac{1}{3k + 3} - \frac{2}{3k + 2} + \frac{4}{3k + 1} \right) = \int_0^1 \frac{1}{x + 2} dx,$$

and evaluate the integral.

(iii) Show that

$$\sum_{k=0}^{\infty} \frac{(-1)^k}{2^{3(k+1)}} \left(\frac{72(2k + 1)}{(3k + 1)(3k + 2)} \right) = \pi\sqrt{a} - \ln b,$$

where a and b are integers which you should determine.



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