STEP II, 2024, Q1 MS

Question	Answer	Mark
1 i	The two sums are $\frac{1}{2}(n+k)(2c+(n+k-1))$ and $\frac{1}{2}n(2(c+n+k)+(n-1))$	B1
	Difference simplifies to $\frac{1}{2}(2ck+k^2-2n^2-k)$	M1
	Two sums are equal if and only if the difference is 0 if and only $2n^2+k=2ck+k^2$	A1
		[3]
ii a	If $k=1$, require $n^2=c$. Any value of n is possible.	B1
b	If $k=2$, require $n^2=2c+1$	M1
	n can be any odd value,	A1
	and $c=\frac{n^2-1}{2}$	A1
		[4]
iii	If $k=4$, require $n^2=4c+6$	B1
	RHS has a factor of 2, but not a factor of 4 …	E1
	… so cannot be a square.	E1
		[3]
iv a	If $c=1$, require $2n^2=k(k+1)$:	
	$k=1, n=1$	B1
	$k=8, n=6$	B1
		[2]

Question	Answer	Mark
1 iv b		
	Since (N, K) is a solution: $2N^2 + K = 2K + K^2$ or $2N^2 = K(K+1)$	B1
	$2(3N + 2K + 1)^2 + (4N + 3K + 1) =$ $18N^2 + 8K^2 + 3 + 24NK + 16N + 11K$ OR $2(3N + 2K + 1)^2 =$ $18N^2 + 8K^2 + 2 + 24NK + 12N + 8K$	M1
	$2(4N + 3K + 1) + (4N + 3K + 1)^2 =$ $16N^2 + 9K^2 + 3 + 24NK + 16N + 12K$ OR $(4N + 3K + 1)(4N + 3K + 2) =$ $16N^2 + 9K^2 + 2 + 24NK + 12N + 9K$	M1
	Difference between the two expressions that use = $2N^2 - K^2 - K$	M1
	$= 0$, so $N' = (3N + 2K + 1)$ is a possible value for n, with $K' = (4N + 3K + 1)$ as the corresponding value of k.	A1
		[5]
c	Use of recurrence with one of the pairs found in part (iv)(a)	M1
	$k = 49, n = 35$	A1
	$k = 288, n = 204$	A1
		[3]