STEP II, 2024, Q10

10 A triangular prism lies on a horizontal plane. One of the rectangular faces of the prism is vertical; the second is horizontal and in contact with the plane; the third, oblique rectangular face makes an angle α with the horizontal. The two triangular faces of the prism are right angled triangles and are vertical. The prism has mass M and it can move without friction across the plane.

A particle of mass m lies on the oblique surface of the prism. The contact between the particle and the plane is rough, with coefficient of friction μ.

(i) Show that if $\mu < \tan\alpha$, then the system cannot be in equilibrium.

Let $\mu = \tan\lambda$, with $0 < \lambda < \alpha < \frac{1}{4}\pi$.

A force P is exerted on the vertical rectangular face of the prism, perpendicular to that face and directed towards the interior of the prism. The particle and prism accelerate, but the particle remains in the same position relative to the prism.

(ii) Show that the magnitude, F, of the frictional force between the particle and the prism is

$$F = \frac{m}{M+m}\left|(M+m)g\sin\alpha - P\cos\alpha\right|.$$

Find a similar expression for the magnitude, N, of the normal reaction between the particle and the prism.

(iii) Hence show that the force P must satisfy

$$(M+m)g\tan(\alpha-\lambda) \leqslant P \leqslant (M+m)g\tan(\alpha+\lambda).$$