

STEP II, 2024, Q1

- 1 In the equality

$$4 + 5 + 6 + 7 + 8 = 9 + 10 + 11,$$

the sum of the five consecutive integers from 4 upwards is equal to the sum of the next three consecutive integers.

Throughout this question, the variables n , k and c represent positive integers.

- (i) Show that the sum of the $n + k$ consecutive integers from c upwards is equal to the sum of the next n consecutive integers if and only if

$$2n^2 + k = 2ck + k^2.$$

- (ii) Find the set of possible values of n , and the corresponding values of c , in each of the cases

(a) $k = 1$

(b) $k = 2$.

- (iii) Show that there are no solutions for c and n if $k = 4$.

- (iv) Consider now the case where $c = 1$.

- (a) Find two possible values of k and the corresponding values of n .

- (b) Show, given a possible value N of n , and the corresponding value K of k , that

$$N' = 3N + 2K + 1$$

will also be a possible value of n , with

$$K' = 4N + 3K + 1$$

as the corresponding value of k .

- (c) Find two further possible values of k and the corresponding values of n .



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