STEP II, 2023, Q9

9 A truck of mass M is connected by a light, rigid tow-bar, which is parallel to the ground, to a trailer of mass kM. A constant driving force D which is parallel to the ground acts on the truck, and the only resistance to motion is a frictional force acting on the trailer, with coefficient of friction μ.

- When the truck pulls the trailer up a slope which makes an angle α to the horizontal, the acceleration is a_1 and there is a tension T_1 in the tow-bar.
- When the truck pulls the trailer on horizontal ground, the acceleration is a_2 and there is a tension T_2 in the tow-bar.
- When the truck pulls the trailer down a slope which makes an angle α to the horizontal, the acceleration is a_3 and there is a tension T_3 in the tow-bar.

All accelerations are taken to be positive when in the direction of motion of the truck.

(i) Show that $T_1 = T_3$ and that $M(a_1 + a_3 - 2a_2) = 2(T_2 - T_1)$.

(ii) It is given that $\mu < 1$.

(a) Show that

$$a_2 < \tfrac{1}{2}(a_1 + a_3) < a_3 \,.$$

(b) Show further that

$$a_1 < a_2 \,.$$