STEP II, 2023, Q7 MS

Question			Answer	Mark
7	(i)		$\|zw\|^2 = \|(ac-bd)+i(ad+bc)\|^2$ $= (ac-bd)^2 + (ad+bc)^2$ $= a^2c^2 + b^2d^2 + a^2d^2 + b^2c^2$	M1
			$\|z\|^2\|w\|^2 = (a^2+b^2)(c^2+d^2)$ $= a^2c^2 + b^2d^2 + a^2d^2 + b^2c^2$	M1
			Therefore $\|zw\|^2 = \|z\|^2\|w\|^2$	A1
				[3]
	(ii)		$\|2+i\| = \sqrt{5}$ and $\|10+11i\| = \sqrt{221}$	B1
			so $9^2 + 32^2 = (2^2 + 1^2)(10^2 + 11^2) = 5 \times 221$	B1
				[2]
	(iii)		8045 = 5 ×1609	M1
			= $(2^2 + 1^2)(40^2 + 3^2)$	M1
			so $\|(2 + i)(40 + 3i)\|^2 = 77^2 + 46^2 = 8045$ (also $34^2 + 83^2$)	A1
				[3]
	(iv)		$612^2 + 1206^2 = 6^2 \times 50805$	B1
				[1]
	(v)		$1002082 = 1001^2 + 9^2$	B1
			so one pair is $5005^2 + 45^2$	A1
			but $25 = 3^2 + 4^2$ and $(3 + 4i)(1001 + 9i)$ $= 2967 + 4031i$	M1
			so a second pair is $2967^2 + 4031^2$	A1
			also, $(4 + 3i)(1001 + 9i) = 3977 + 3039i$	M1
			so a third pair is $3977^2 + 3039^2$	A1
				[6]
	(vi)		require $(10^2 + 3^2)(c^2 + d^2) = (1001^2 + 6^2)$	M1
			implies simultaneous equations for c and d	M1
			$10c - 3d = 1001$, $10d + 3c = 6$ or $3c - 10d = 1001$, $3d + 10c = 6$	A1
			giving $c = 92$, $d = -27$ (from the first set)	A1
			so $9193 = 92^2 + 27^2$ (also $38^2 + 211^2$)	A1
				[5]