

STEP II, 2023, Q7 MS

Question		Answer	Mark
7	(i)	$ zw ^2 = (ac - bd) + i(ad + bc) ^2$	M1
		$= (ac - bd)^2 + (ad + bc)^2$	
		$= a^2c^2 + b^2d^2 + a^2d^2 + b^2c^2$	
		$ z ^2 w ^2 = (a^2 + b^2)(c^2 + d^2)$	M1
		$= a^2c^2 + b^2d^2 + a^2d^2 + b^2c^2$	
		Therefore $ zw ^2 = z ^2 w ^2$	A1
			[3]
	(ii)	$ 2 + i = \sqrt{5}$ and $ 10 + 11i = \sqrt{221}$	B1
		so $9^2 + 32^2 = (2^2 + 1^2)(10^2 + 11^2) = 5 \times 221$	B1
			[2]
	(iii)	$8045 = 5 \times 1609$	M1
		$= (2^2 + 1^2)(40^2 + 3^2)$	M1
		so $ (2 + i)(40 + 3i) ^2 = 77^2 + 46^2 = 8045$ (also $34^2 + 83^2$)	A1
			[3]
	(iv)	$612^2 + 1206^2 = 6^2 \times 50805$	B1
			[1]
	(v)	$1002082 = 1001^2 + 9^2$	B1
		so one pair is $5005^2 + 45^2$	A1
		but $25 = 3^2 + 4^2$ and $(3 + 4i)(1001 + 9i)$ $= 2967 + 4031i$	M1
		so a second pair is $2967^2 + 4031^2$	A1
		also, $(4 + 3i)(1001 + 9i) = 3977 + 3039i$	M1
		so a third pair is $3977^2 + 3039^2$	A1
			[6]
	(vi)	require $(10^2 + 3^2)(c^2 + d^2) = (1001^2 + 6^2)$	M1
		implies simultaneous equations for c and d	M1
		$10c - 3d = 1001, 10d + 3c = 6$	A1
		or	
		$3c - 10d = 1001, 3d + 10c = 6$	
		giving $c = 92, d = -27$ (from the first set)	A1
		so $9193 = 92^2 + 27^2$ (also $38^2 + 211^2$)	A1
			[5]



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