

STEP II, 2023, Q7

- 7 (i) The complex numbers z and w have real and imaginary parts given by $z = a + ib$ and $w = c + id$. Prove that $|zw| = |z||w|$.
- (ii) By considering the complex numbers $2 + i$ and $10 + 11i$, find positive integers h and k such that $h^2 + k^2 = 5 \times 221$.
- (iii) Find positive integers m and n such that $m^2 + n^2 = 8045$.
- (iv) You are given that $102^2 + 201^2 = 50805$.
Find positive integers p and q such that $p^2 + q^2 = 36 \times 50805$.
- (v) Find three distinct pairs of positive integers r and s such that $r^2 + s^2 = 25 \times 1002082$ and $r < s$.
- (vi) You are given that $109 \times 9193 = 1002037$.
Find positive integers t and u such that $t^2 + u^2 = 9193$.



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