

STEP II, 2023, Q6 MS

Question	Answer	Mark
6	Induction structure	M1
	Base case	B1
	$\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} F_{n+1} & F_n \\ F_n & F_{n-1} \end{pmatrix} = \begin{pmatrix} F_{n+1} + F_n & F_n + F_{n-1} \\ F_{n+1} & F_n \end{pmatrix}$ or $\begin{pmatrix} F_{n+1} & F_n \\ F_n & F_{n-1} \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} F_{n+1} + F_n & F_{n+1} \\ F_n + F_{n-1} & F_n \end{pmatrix}$	A1
	Use of definition (of F_n) and conclusion (of induction)	A1
		[4]
(i)	Use of $\det(Q^n) = (\det Q)^n$	M1
	clearly shown	A1
		[2]
(ii)	Use of (1,2) entry in $Q^{m \times n} = Q^m Q^n$	M1
	clearly shown	A1
		[2]
(iii)	$Q^2 = Q + I$	B1
		[1]
(a)	Use of $Q^{2^n} = (Q + I)^n$	M1
	and Binomial expansion	M1
	clearly shown	A1
		[3]
(b)	Derivation of $Q^3 = Q(Q + I) = 2Q + I$ (give the mark for any one of these)	B1
	Use of $Q^{3^n} = (2Q + I)^n$ and Binomial expansion	M1
	clearly shown	A1
	Use of $Q^{3^n} = Q^n(Q + I)^n$ and Binomial expansion	M1
	clearly shown	A1
		[5]
(c)	Use of $I = Q^n(Q - I)^n$ (or $(-Q)^n (I - Q)^n$)	M1
	Use of binomial expansion	M1
	clearly shown	A1
		[3]



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