

STEP II, 2023, Q5 EC

Question 5

In part (i) (a) most candidates realised that induction was necessary. Although “explain briefly” was written in the question, some candidates omitted necessary components of an inductive argument here. Some candidates incorrectly stated that the sequence always increased. A popular alternative method was stating $x_{n+2} > x_{n+1}$. In this case it is necessary to observe that the denominator is positive to secure full marks.

In part (i) (b) many candidates were successful here in rewriting $x_{n+1}^2 - 2$ in terms of x_n but some failed to assert (and very briefly justify) the strict positivity of $(x_n + 1)^2$ in order to show that $x_{n+1}^2 - 2$ and $x_n^2 - 2$ have opposite signs. When showing $|x_{n+1}^2 - 2| \leq |x_n^2 - 2|/4$ the most common mistake was to not use absolute value signs, and write false assertions like $x_{n+1}^2 - 2 \leq (x_n^2 - 2)/4$, which is false for odd n .

In part (i) (c) many students used the inequality in the previous part repeatedly to write $|x_{10}^2 - 2| \leq |x_0^2 - 2|/4^{10}$ but did not give a justification that $4^{10} > 10^6$. A small number of candidates were able to calculate x_{10} , and x_{10}^2 successfully and numerically compare these to 2 and $2 - 10^{-6}$, however almost all attempts at this were unsuccessful.

Almost all candidates who attempted part (ii) (a) earned at least one mark. In several cases candidates did not formulate a standard inductive argument, either missing the base case or not using an inductive hypothesis.

In part (ii) (b) many candidates used $n=0$ as a base case, but this is not valid here. Of those who opted for an alternative method of using recursion to write $y_n - \sqrt{2}$ in terms of $y_0 - \sqrt{2}$, few were able to justify the exponent for powers of 2. Candidates who attempted a full inductive proof often earned at least 2 of the 4 marks for this part.

Candidates attempting part (ii)(c) often earned some marks for showing the correct method, but errors in the accuracy of the work meant that few were able to achieve full marks here.



NextStepMaths.com

To view mark schemes, fully worked solutions and examiner's comments, and for more details about tutoring and other services offered, go to [NextStepMaths.com](https://www.NextStepMaths.com)