

## STEP II, 2023, Q4

- 4 (i) Show that, if  $(x - \sqrt{2})^2 = 3$ , then  $x^4 - 10x^2 + 1 = 0$ .  
Deduce that, if  $f(x) = x^4 - 10x^2 + 1$ , then  $f(\sqrt{2} + \sqrt{3}) = 0$ .
- (ii) Find a polynomial  $g$  of degree 8 with integer coefficients such that  $g(\sqrt{2} + \sqrt{3} + \sqrt{5}) = 0$ .  
Write your answer in a form without brackets.
- (iii) Let  $a$ ,  $b$  and  $c$  be the three roots of  $t^3 - 3t + 1 = 0$ .  
Find a polynomial  $h$  of degree 6 with integer coefficients such that  $h(a + \sqrt{2}) = 0$ ,  
 $h(b + \sqrt{2}) = 0$  and  $h(c + \sqrt{2}) = 0$ . Write your answer in a form without brackets.
- (iv) Find a polynomial  $k$  with integer coefficients such that  $k(\sqrt[3]{2} + \sqrt[3]{3}) = 0$ . Write your answer in a form without brackets.



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