STEP II, 2023, Q3 EC

Question 3

In part (i) (a) when assuming that the degree of p is odd for a contradiction, many also assumed that the lead coefficient of p(x) was positive and so made the statement that p(x) tends to minus infinity as x tends to minus infinity which is not necessarily correct (unless an argument that the lead coefficient is positive was provided). Many candidates did not provide sufficient detail and so were not awarded full marks for this part.

In part (i)(b) candidates generally produced good answers, but a number lost marks for not stating that the $(n+1)^{\text{th}}$ derivative of p is zero sufficiently clearly. Some used +... at the end of the sum of polynomials that define q(x) or just didn't discuss the final term of q'(x) and again in these cases it was not sufficiently clear that the key idea had been understood.

In part (ii) (a) candidates generally completed the first part well, but a significant number of candidates lost a mark because their argument was the wrong way round, arguing that B implies A rather than A implies B. A significant number of candidates realised that all the stationary points of q must have a positive y-coordinate but they didn't link this to q(x) being positive for large $|x|$ to get all the marks.

In part (ii) (b) the first part was again usually very well done. In a similar way to part (a) there were a good number of impressive answers to 'q(x)>0 for all x' but many lost marks by not providing sufficient detail or not including all aspects of the argument (particularly that q(x)>0 for large x).

Part (ii) (c) was generally very well done. Virtually all candidates used the right method for the first part but some lost a mark for not providing sufficient detail in the substitution in integration by parts. Most did the rest of this part well but quite a few candidates lost marks for not dealing with the end term of the summation correctly - in the main line of the solution it is an integral which candidates should explain is zero. Some neglected to include this term without comment or used +... at the end of the sum and, in these cases, it was not clear that the idea had been understood.