

STEP II, 2023, Q2

- 2 (i) The real numbers x , y and z satisfy the equations

$$y = \frac{2x}{1-x^2},$$

$$z = \frac{2y}{1-y^2},$$

$$x = \frac{2z}{1-z^2}.$$

Let $x = \tan \alpha$. Deduce that $y = \tan 2\alpha$ and show that $\tan \alpha = \tan 8\alpha$.

Find all solutions of the equations, giving each value of x , y and z in the form $\tan \theta$ where $-\frac{1}{2}\pi < \theta < \frac{1}{2}\pi$.

- (ii) Determine the number of real solutions of the simultaneous equations

$$y = \frac{3x - x^3}{1 - 3x^2},$$

$$z = \frac{3y - y^3}{1 - 3y^2},$$

$$x = \frac{3z - z^3}{1 - 3z^2}.$$

- (iii) Consider the simultaneous equations

$$y = 2x^2 - 1,$$

$$z = 2y^2 - 1,$$

$$x = 2z^2 - 1.$$

- (a) Determine the number of real solutions of these simultaneous equations with $|x| \leq 1$, $|y| \leq 1$, $|z| \leq 1$.
- (b) By finding the degree of a single polynomial equation which is satisfied by x , show that all solutions of these simultaneous equations have $|x| \leq 1$, $|y| \leq 1$, $|z| \leq 1$.



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