

STEP II, 2022, Q2 MS

Question	Answer	Mark
2 (i)	$u_{n+2} - u_{n+1} = u_{n+1} - u_n$	M1
	so constant differences.	A1
	If $u_n - u_{n-1} = d$, then $u_n = u_1 + (n-1)d$ which is of degree at most 1	B1
		[3]
(ii)	$t_{n+1} + p(n+1)^2$ $= \frac{1}{2}(t_{n+2} + p(n+2)^2 + t_n + pn^2) - p$	M1
	so $t_{n+1} = \frac{1}{2}(t_{n+2} + t_n)$	A1
	so t_n has degree at most 1	A1
	Hence since $p \neq 0$, v_n has degree 2.	A1
	Taking $v_n = pn^2 + qn + r$, gives: $p + q + r = 0$ $4p + 2q + r = 0$	M1
	so $q = -3p$	A1
	And $r = 2p$	A1
		[7]
(iii)	Substitutes $w_n = t_n + kn^3$, so	B1
	$t_{n+1} + k(n+1)^3$ $= \frac{1}{2}(t_{n+2} + k(n+2)^3 - t_n + kn^3) - an - b$	M1
	LHS and RHS both give $kn^3 + 3kn^2$ terms	A1
	$t_{n+1} = \frac{1}{2}(t_{n+2} + t_n) + (3k - a)n - (b - 3k)$	A1
	Choosing $k = \frac{1}{3}a$	A1
	gives case (ii) (with $p = b - a$) so t_n has degree at most 2 and w_n has degree 3, as $a \neq 0$.	A1
	unless $b = a$, when case (i) applies so t_n has degree at most 2 and w_n has degree 3, as $a \neq 0$.	A1
	Taking $w_n = \frac{1}{3}an^3 + (b-a)n^2 + qn + r$ gives $b - \frac{2}{3}a + q + r = 0$ $-\frac{4}{3}a + 4b + 2q + r = 0$	M1
	so $q = \frac{2}{3}a - 3b$	A1
	and $r = 2b$	A1
	$w_n = \frac{1}{3}an^3 + (b-a)n^2 + (\frac{2}{3}a - 3b)n + 2b$	
		[10]



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