# STEP II, 2022, Q2 MS

| Question | | | Answer | Mark |
|---|---|---|---|---|
| 2 | (i) | | $u_{n+2} - u_{n+1} = u_{n+1} - u_n$ | M1 |
| | | | so constant differences. | A1 |
| | | | If $u_n - u_{n-1} = d$, then $u_n = u_1 + (n-1)d$ which is of degree at most 1 | B1 |
| | | | | [3] |
| | (ii) | | $t_{n+1} + p(n+1)^2 = \frac{1}{2}(t_{n+2} + p(n+2)^2 + t_n + pn^2) - p$ | M1 |
| | | | so $t_{n+1} = \frac{1}{2}(t_{n+2} + t_n)$ | A1 |
| | | | so $t_n$ has degree at most 1 | A1 |
| | | | Hence since $p \neq 0$, $v_n$ has degree 2. | A1 |
| | | | Taking $v_n = pn^2 + qn + r$, gives: $p + q + r = 0$ $4p + 2q + r = 0$ | M1 |
| | | | so $q = -3p$ | A1 |
| | | | And $r = 2p$ | A1 |
| | | | | [7] |
| | (iii) | | Substitutes $w_n = t_n + kn^3$, so | B1 |
| | | | $t_{n+1} + k(n+1)^3 = \frac{1}{2}(t_{n+2} + k(n+2)^3 - t_n + kn^3) - an - b$ | M1 |
| | | | LHS and RHS both give $kn^3 + 3kn^2$ terms | A1 |
| | | | $t_{n+1} = \frac{1}{2}(t_{n+2} + t_n) + (3k - a)n - (b - 3k)$ | A1 |
| | | | Choosing $k = \frac{1}{3}a$ | A1 |
| | | | gives case **(ii)** (with $p = b - a$) so $t_n$ has degree at most 2 and $w_n$ has degree 3, as $a \neq 0$. | A1 |
| | | | unless $b = a$, when case **(i)** applies so $t_n$ has degree at most 2 and $w_n$ has degree 3, as $a \neq 0$. | A1 |
| | | | Taking $w_n = \frac{1}{3}an^3 + (b-a)n^2 + qn + r$ gives $b - \frac{2}{3}a + q + r = 0$ $-\frac{4}{3}a + 4b + 2q + r = 0$ | M1 |
| | | | so $q = \frac{2}{3}a - 3b$ | A1 |
| | | | and $r = 2b$ | A1 |
| | | | $w_n = \frac{1}{3}an^3 + (b-a)n^2 + \left(\frac{2}{3}a - 3b\right)n + 2b$ | |
| | | | | [10] |