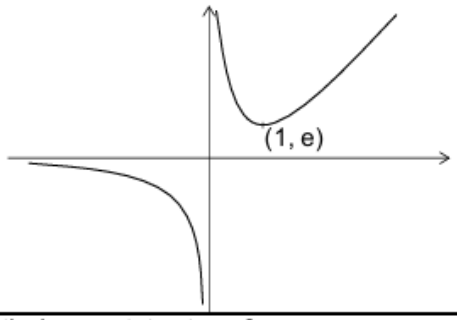
STEP II, 2022, Q1 MS

Question			Answer	Mark
1	(i)		$\int \frac{3x^3}{\sqrt{1+x^3}}\,dx = u\cdot v - \int u'v\,dx$	M1
			$\int \frac{3x^3}{\sqrt{1+x^3}}\,dx = x\cdot k\sqrt{1+x^3} - \int k\sqrt{1+x^3}\,dx$	M1
			$\int \frac{3x^3}{\sqrt{1+x^3}}\,dx = x\cdot 2\sqrt{1+x^3} - \int 2\sqrt{1+x^3}\,dx$	A1
			so $\int 2\sqrt{1+x^3} + \frac{3x^3}{\sqrt{1+x^3}}\,dx = 2x\sqrt{1+x^3} + c$	A1
				[4]
	(ii)		$\frac{(x^2+2)\sin x}{x^3} = \frac{\sin x}{x} + \frac{2\sin x}{x^3}$	M1
			$\int \frac{2\sin x}{x^3}\,dx = -\frac{p}{x^2}\cdot\sin x + \int \frac{q\cos x}{x^2}\,dx$	M1
			$= -\frac{p}{x^2}\cdot\sin x - \frac{r}{x}\cdot\cos x - \int \frac{s\sin x}{x}\,dx$	M1
			$-\frac{1}{x^2}\cdot\sin x - \frac{1}{x}\cdot\cos x - \int \frac{\sin x}{x}\,dx$	
			so $\int \left(x^2+2\right)\frac{\sin x}{x^3}\,dx = -\frac{\sin x + x\cos x}{x^2} + c$	A1
				[4]
	(iii)	(a)	$\frac{dy}{dx} = \frac{(x-1)e^x}{x^2}$	M1
			Therefore there is a stationary point at (1, e).	A1
				
			Vertical asymptote at $x = 0$	G1
			Minimum in first quadrant and correct behaviour as $x \to \infty$	G1
			Correct behaviour as $x \to -\infty$	G1
				[5]

		(b)	$\int_a^{2a} \frac{e^x}{x^2}\,dx = \left[-\frac{p}{x}\cdot e^x\right]_a^{2a} + \int_a^{2a} \frac{qe^x}{x}\,dx$	**M1**
			$\int_a^{2a} \frac{e^x}{x^2}\,dx = \left[-\frac{1}{x}\cdot e^x\right]_a^{2a} + \int_a^{2a} \frac{e^x}{x}\,dx$	**A1**
			Therefore for integrals to be equal we need $\left[-\frac{1}{x}\cdot e^x\right]_a^{2a} = 0$	**M1**
			$-\frac{1}{2a}\cdot e^{2a} + \frac{1}{a}\cdot e^a = 0$ $\frac{1}{2a}\cdot e^a\left(-e^a + 2\right) = 0$	**M1**
			so $a = \ln 2$	**A1**
				[5]
		(c)	As before, this means we would need $\left[-\frac{1}{x}\cdot e^x\right]_m^n$ i.e. $\frac{e^n}{n} = \frac{e^m}{m}$	**B1**
			From the graph in part **(iii) (a)** this would mean that the smaller of n, m must lie in the range (0, 1). Hence this is not an integer.	**E1**
				[2]