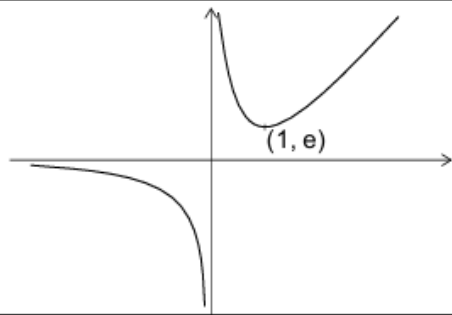


STEP II, 2022, Q1 MS

Question		Answer	Mark
1	(i)	$\int \frac{3x^3}{\sqrt{1+x^3}} dx = u \cdot v - \int u'v dx$	M1
		$\int \frac{3x^3}{\sqrt{1+x^3}} dx = x \cdot k\sqrt{1+x^3} - \int k\sqrt{1+x^3} dx$	M1
		$\int \frac{3x^3}{\sqrt{1+x^3}} dx = x \cdot 2\sqrt{1+x^3} - \int 2\sqrt{1+x^3} dx$	A1
		so $\int 2\sqrt{1+x^3} + \frac{3x^3}{\sqrt{1+x^3}} dx = 2x\sqrt{1+x^3} + c$	A1
			[4]
	(ii)	$\frac{(x^2+2)\sin x}{x^3} = \frac{\sin x}{x} + \frac{2\sin x}{x^3}$	M1
		$\int \frac{2\sin x}{x^3} dx = -\frac{p}{x^2} \cdot \sin x + \int \frac{q \cos x}{x^2} dx$	M1
		$= -\frac{p}{x^2} \cdot \sin x - \frac{r}{x} \cdot \cos x - \int \frac{s \sin x}{x} dx$	M1
		$-\frac{1}{x^2} \cdot \sin x - \frac{1}{x} \cdot \cos x - \int \frac{\sin x}{x} dx$	
		so $\int (x^2+2) \frac{\sin x}{x^3} dx = -\frac{\sin x + x \cos x}{x^2} + c$	A1
			[4]
	(iii) (a)	$\frac{dy}{dx} = \frac{(x-1)e^x}{x^2}$	M1
		Therefore there is a stationary point at (1, e).	A1
			
		Vertical asymptote at $x = 0$	G1
		Minimum in first quadrant and correct behaviour as $x \rightarrow \infty$	G1
		Correct behaviour as $x \rightarrow -\infty$	G1
			[5]



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	(b)	$\int_a^{2a} \frac{e^x}{x^2} dx = \left[-\frac{p}{x} \cdot e^x \right]_a^{2a} + \int_a^{2a} \frac{qe^x}{x} dx$	M1
		$\int_a^{2a} \frac{e^x}{x^2} dx = \left[-\frac{1}{x} \cdot e^x \right]_a^{2a} + \int_a^{2a} \frac{e^x}{x} dx$	A1
		Therefore for integrals to be equal we need $\left[-\frac{1}{x} \cdot e^x \right]_a^{2a} = 0$	M1
		$-\frac{1}{2a} \cdot e^{2a} + \frac{1}{a} \cdot e^a = 0$ $\frac{1}{2a} \cdot e^a (-e^a + 2) = 0$	M1
		so $a = \ln 2$	A1
			[5]
	(c)	As before, this means we would need $\left[-\frac{1}{x} \cdot e^x \right]_m^n$ i.e. $\frac{e^n}{n} = \frac{e^m}{m}$	B1
		From the graph in part (iii) (a) this would mean that the smaller of n, m must lie in the range $(0, 1)$. Hence this is not an integer.	E1
			[2]



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