STEP II, 2022, Q11 MS

Question			Answer	Mark
11	(i)		Expected net loss is $q^T(\cdots)$	M1
			$(\cdots(1-q^{N-T})-\cdots q^{N-T})$	M1
			$=q^T(D(1-q^{N-T})-(N-T)q^{N-T})$	A1
				[3]
	(ii)		all variables non-negative and $N \geq T$, $D>0$, so denominator positive so $\alpha \geq 0$.	B1
			$N(N-T+D)-DT=(N+D)(N-T)>0$, so $\alpha<1$	B1
			$\frac{d}{dq}[expected\ net\ loss]=0$	M1
			$TDq^{T-1}-N(N-T+D)q^{N-1}=0$	A1
			$N(N-T+D)q^{T-1}(\alpha-q^{N-T})=0$	A1
			hence $q=\alpha^{\frac{1}{N-T}}$ determines exactly one value of q with $0 \leqslant q<1$ for which the expected net loss is stationary	A1
			$\frac{d^2}{dq^2}[expected\ net\ loss]$ $=T(T-1)Dq^{T-2}-N(N-1)(N-T+D)q^{N-2}$	M1
			at the root $=N(N-T+D)q^{T-2}\left((T-1)\alpha-(N-1)q^{N-T}\right)$	M1
			$=N(N-T+D)q^{N-2}\left((T-1)-(N-1)\right)$ at the root	M1
			but $-N(N-T)(N-T+D)q^{N-2}<0$, so maximum	A1
			The maximum net loss is $q^T\left(D-\alpha(N-T+D)\right)$	M1
			$=\frac{Dq^T}{N}(N-T)$ but $q^T=(q^{N-T})^{\frac{T}{N-T}}=\alpha^k$	A1
				[12]
	(iii)		The expected loss is an increasing function of T if the expected net loss with one extra stick tested is larger than that without the extra stick	M1
			so when $[Dq^{T+1}-q^N(N-(T+1)+D)]-[Dq^T-q^N(N-T+D)]$ $=q^T(q^{N-T}-Dp)>0$ for all T	A1
			which is the case if $q^{N-T}>Dp$	A1
			As p tends to zero, the left hand side of this expression tends to 1, and the right hand side to 0 hence there exists $\beta>0$ such that, for all $p<\beta$, the expected net loss is an increasing function of T	E1
			Thus for $p<\beta$, testing no sticks minimises the expected net loss.	E1
				[5]