STEP II, 2021, Q9 MS

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(i)

(a) The forces acting on the particle at P are:

$W = Mg$ (directed downwards) **M1**

$T_1 = m_1 g$ (directed towards Q) **A1**

$T_2 = m_2 g$ (directed towards R)

By the triangle inequality: **dM1**

$Mg < m_1 g + m_2 g$

$M < m_1 + m_2$ **A1**

$T_1^2 = T_2^2 + W^2 - 2T_2 W \cos\theta_2$

Since θ_2 is acute $\cos\theta_2 > 0$, so **M1**

$T_1^2 < T_2^2 + W^2$

$M^2 g^2 > m_1^2 g^2 - m_2^2 g^2$ **E1**

$\sqrt{m_1^2 - m_2^2} < M$ **A1**

(b) $QS = PS\tan\theta_1$ and $SR = PS\tan\theta_2$

If S divides QR in the ratio $r:1$, then $QS = rSR$

$$r = \frac{\tan\theta_1}{\tan\theta_2}$$ **M1**

By the sine rule:

$$\frac{\sin\theta_2}{m_1 g} = \frac{\sin\theta_1}{m_2 g}$$ **M1**

By the cosine rule:

$$\cos\theta_1 = \frac{T_1^2 + W^2 - T_2^2}{2T_1 W} = \frac{m_1^2 + M^2 - m_2^2}{2m_1 M}$$ **M1**

Similarly:

$$\cos\theta_2 = \frac{m_2^2 + M^2 - m_1^2}{2m_2 M}$$ **M1**

Therefore:

$$r = \frac{\sin\theta_1}{\sin\theta_2} \times \frac{\cos\theta_2}{\cos\theta_1}$$

$$= \frac{m_2}{m_1} \times \frac{\frac{m_2^2 + M^2 - m_1^2}{2m_2 M}}{\frac{m_1^2 + M^2 - m_2^2}{2m_1 M}} = \frac{m_2^2 + M^2 - m_1^2}{m_1^2 + M^2 - m_2^2}$$ ***AG*** **dM1** **A1**

(ii) From the triangle of forces, the angle between T_1 and T_2 must be 90° (Pythagoras)

Therefore $\theta_1 + \theta_2 = 90°$ **B1**

By (i)(b)

$$r = \frac{m_2^2}{m_1^2}$$ **M1**

Let d be such that $QS = m_2^2 d$ and $SR = m_1^2 d$. **M1**

Since triangles PSQ and RSP are similar: **M1**

$$\frac{SP}{QS} = \frac{RS}{SP}$$ **M1**

$$PS^2 = m_1^2 m_2^2 d^2$$ **A1**

Therefore, $SP = m_1 m_2 d$ and QR= $(m_1^2 + m_2^2)d$, so the ratio of QR to SP is:

$$M^2 : m_1 m_2$$ **A1**