

STEP II, 2021, Q7

7 (i) The matrix \mathbf{R} represents an anticlockwise rotation through angle ϕ ($0^\circ \leq \phi < 360^\circ$) in two dimensions, and the matrix $\mathbf{R} + \mathbf{I}$ also represents a rotation in two dimensions. Determine the possible values of ϕ and deduce that $\mathbf{R}^3 = \mathbf{I}$.

(ii) Let \mathbf{S} be a real matrix with $\mathbf{S}^3 = \mathbf{I}$, but $\mathbf{S} \neq \mathbf{I}$.

Show that $\det(\mathbf{S}) = 1$.

Given that

$$\mathbf{S} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

show that $\mathbf{S}^2 = (a + d)\mathbf{S} - \mathbf{I}$.

Hence prove that $a + d = -1$.

(iii) Let \mathbf{S} be a real 2×2 matrix.

Show that if $\mathbf{S}^3 = \mathbf{I}$ and $\mathbf{S} + \mathbf{I}$ represents a rotation, then \mathbf{S} also represents a rotation. What are the possible angles of the rotation represented by \mathbf{S} ?



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