

STEP II, 2021, Q4 MS

4

(i) $\frac{dy}{dx} = xe^x + e^x$ **M1**

Since $e^x > 0$ for all x , the only stationary point is when $x = -1$ **A1**
 Coordinates of stationary point are $(-1, -\frac{1}{e})$

Sketch showing:

$y \rightarrow \infty$ as $x \rightarrow \infty$ and $y \rightarrow 0^-$ as $x \rightarrow -\infty$ **G1**

Curve passing through $(0,0)$ with stationary point at $(-1, -\frac{1}{e})$ indicated. **G1**

(ii) -1 **B1**

Sketch showing reflection of the correct portion of the graph in the line $y = x$. **G1**
 domain $[-\frac{1}{e}, \infty)$ and range $[-1, \infty)$

(iii)

(a) $e^{-x} = 5x$ **M1**
 $xe^x = \frac{1}{5}$

$f(x) = \frac{1}{5}$ **A1**
 Since $f(x) > 0$ there is only one solution

$$x = g\left(\frac{1}{5}\right)$$

(b) $2x \ln x + 1 = 0$ **M1**
 Let $u = \ln x$: **M1**

$$ue^u = -\frac{1}{2}$$

The minimum value of $f(x)$ is $-\frac{1}{e}$ and $-\frac{1}{2} < -\frac{1}{e}$, so there are no solutions. **E1**



NextStepMaths.com

To view mark schemes, fully worked solutions and
 examiner's comments, and for more details about
 tutoring and other services offered, go to
NextStepMaths.com

(c) $3x \ln x + 1 = 0$

Let $u = \ln x$:

$$ue^u = -\frac{1}{3} \quad \text{M1}$$

$-\frac{1}{e} < -\frac{1}{3} < 0$ so there are two solutions for u and the greater of the two will be **E1**
when $u = g\left(-\frac{1}{3}\right)$.

$x = e^{g\left(-\frac{1}{3}\right)}$ is the larger value. **A1**

(d) $x = 3 \ln x$

Let $u = \ln x$:

$$ue^{-u} = \frac{1}{3} \quad \text{M1}$$

$(-u)e^{-u} = -\frac{1}{3}$, so (as in (c)) $g\left(-\frac{1}{3}\right)$ is the greater of the two possible values for $-u$. **M1**

Therefore $x = e^{-g\left(-\frac{1}{3}\right)}$ is the smaller value. **A1**

E1

(iv) $x \ln x = \ln 10$

Let $u = \ln x$:

$$ue^u = \ln 10 \quad \text{M1}$$

$$u = g(\ln 10)$$

$$x = e^{g(\ln 10)} \quad \text{A1}$$



NextStepMaths.com

To view mark schemes, fully worked solutions and examiner's comments, and for more details about tutoring and other services offered, go to [NextStepMaths.com](https://www.NextStepMaths.com)