STEP II, 2021, Q4 MS

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(i)	$\frac{dy}{dx} = xe^x + e^x$	**M1**
	Since $e^x > 0$ for all x, the only stationary point is when $x = -1$ Coordinates of stationary point are $(-1, -\frac{1}{e})$	**A1**
	Sketch showing: $y \to \infty$ as $x \to \infty$ and $y \to 0^-$ as $x \to -\infty$ Curve passing through $(0,0)$ with stationary point at $(-1, -\frac{1}{e})$ indicated.	**G1** **G1**
(ii)	-1	**B1**
	Sketch showing reflection of the correct portion of the graph in the line $y = x$. domain $\left[-\frac{1}{e}, \infty\right)$ and range $[-1, \infty)$	**G1**
(iii) (a)	$e^{-x} = 5x$ $xe^x = \frac{1}{5}$	
	$f(x) = \frac{1}{5}$	**M1**
	Since $f(x) > 0$ there is only one solution	**A1**
	$x = g\left(\frac{1}{5}\right)$	
(b)	$2x \ln x + 1 = 0$ Let $u = \ln x$:	**M1**
	$$ue^u = -\frac{1}{2}$$	**M1**
	The minimum value of $f(x)$ is $-\frac{1}{e}$ and $-\frac{1}{2} < -\frac{1}{e}$, so there are no solutions.	**E1**

(c) $3x\ln x + 1 = 0$

Let $u = \ln x$:

$$ue^u = -\frac{1}{3}$$ **M1**

$-\frac{1}{e} < -\frac{1}{3} < 0$ so there are two solutions for u and the greater of the two will be when $u = g\left(-\frac{1}{3}\right)$. **E1**

$x = e^{g\left(-\frac{1}{3}\right)}$ is the larger value. **A1**

(d) $x = 3\ln x$

Let $u = \ln x$:

$$ue^{-u} = \frac{1}{3}$$ **M1**

$(-u)e^{-u} = -\frac{1}{3}$, so (as in (c)) $g\left(-\frac{1}{3}\right)$ is the greater of the two possible values for $-u$. **M1**

Therefore $x = e^{-g\left(-\frac{1}{3}\right)}$ is the smaller value. **A1**

E1

(iv) $x\ln x = \ln 10$

Let $u = \ln x$:

$$ue^u = \ln 10$$ **M1**

$u = g(\ln 10)$

$$x = e^{g(\ln 10)}$$ **A1**