STEP II, 2021, Q2 MS

2 (i)	$3pq - p^3 = 3(a+b)(a^2+b^2) - (a+b)^3$	M1
	$= 2a^3 + 2b^3$	
	$= 2r$ ***AG***	A1
(ii)	$2x^2 - 2px + (p^2 - q) = 0$	
	The roots of the equation a and b satisfy:	M1
	$a + b = p$	B1
	$2ab = p^2 - q$	B1
	$a^2 + b^2 = (a+b)^2 - 2ab$	B1
	$= p^2 - (p^2 - q) = q$	
	$a^3 + b^3 = (a+b)^3 - 3ab(a+b)$	
	$= p^3 - \frac{3}{2}(p^2 - q)p$	
	$= \frac{1}{2}(3pq - p^3) = r$	B1
	So the three equations hold.	E1
(iii)	$a + b = s - c\ (= p)$	
	$a^2 + b^2 = t - c^2\ (= q)$	
	$a^3 + b^3 = u - c^3\ (= r)$	M1
	By part (i):	
	$3(s-c)(t-c^2) - (s-c)^3 = 2(u - c^3)$	
	$3st - 3ct - 3c^2s + 3c^3 - s^3 + 3cs^2 - 3c^2s + c^3 = 2u - 2c^3$	M1
	$6c^3 - 6sc^2 + 3(s^2 - t)c + 3st - s^3 - 2u = 0$	A1
	Therefore c is a root of the equation	
	$6x^3 - 6sx^2 + 3(s^2 - t)x + 3st - s^3 - 2u = 0$ ***AG***	E1
	The other roots are a and b.	B1
	The constant term is $-6 \times$ the product of the roots:	M1
	$-6abc = 3st - s^3 - 2u$	
	$s^3 - 3st + 2u = 6v$ ***AG***	A1
(iv)	By (iii) a, b and c are the roots of	M1
	$6x^3 - 18x^2 + 24x - 12 = 0$	A1
	$6(x-1)(x^2 - 2x + 2) = 0$	M1
	$1, 1+i, 1-i$	A1
	$1 + (1+i) + (1-i) = 3$	
	$1^2 + (1+i)^2 + (1-i)^2 = 1 + (1 + 2i - 1) + (1 - 2i - 1) = 1$	
	$1^3 + (1+i)^3 + (1-i)^3 = 1 + (-2 + 2i) + (-2 - 2i) = -3$	B1
	$1(1+i)(1-i) = 2$	