

STEP II, 2021, Q2 MS

- 2
- (i)
$$3pq - p^3 = 3(a+b)(a^2 + b^2) - (a+b)^3$$

$$= 2a^3 + 2b^3$$

$$= 2r \quad \mathbf{AG}$$
M1
A1
- (ii)
$$2x^2 - 2px + (p^2 - q) = 0$$
 The roots of the equation a and b satisfy:

$$a + b = p$$

$$2ab = p^2 - q$$

$$a^2 + b^2 = (a+b)^2 - 2ab$$

$$= p^2 - (p^2 - q) = q$$

$$a^3 + b^3 = (a+b)^3 - 3ab(a+b)$$

$$= p^3 - \frac{3}{2}(p^2 - q)p$$

$$= \frac{1}{2}(3pq - p^3) = r$$
M1
B1
B1
B1
B1
- So the three equations hold. E1
- (iii)
$$a + b = s - c (= p)$$

$$a^2 + b^2 = t - c^2 (= q)$$

$$a^3 + b^3 = u - c^3 (= r)$$
M1
- By part (i):
- $$3(s-c)(t-c^2) - (s-c)^3 = 2(u-c^3)$$
- $$3st - 3ct - 3c^2s + 3c^3 - s^3 + 3cs^2 - 3c^2s + c^3 = 2u - 2c^3$$
- $$6c^3 - 6sc^2 + 3(s^2 - t)c + 3st - s^3 - 2u = 0$$
- M1
A1
- Therefore c is a root of the equation
- $$6x^3 - 6sx^2 + 3(s^2 - t)x + 3st - s^3 - 2u = 0 \quad \mathbf{AG}$$
- E1
- The other roots are a and b . B1
- The constant term is $-6 \times$ the product of the roots: M1
- $$-6abc = 3st - s^3 - 2u$$
- $$s^3 - 3st + 2u = 6v \quad \mathbf{AG}$$
- A1
- (iv) By (iii) a, b and c are the roots of M1
- $$6x^3 - 18x^2 + 24x - 12 = 0$$
- A1
- $$6(x-1)(x^2 - 2x + 2) = 0$$
- M1
- $$1, 1+i, 1-i$$
- A1
- $1 + (1+i) + (1-i) = 3$
- $1^2 + (1+i)^2 + (1-i)^2 = 1 + (1+2i-1) + (1-2i-1) = 1$
- $1^3 + (1+i)^3 + (1-i)^3 = 1 + (-2+2i) + (-2-2i) = -3$ B1
- $1(1+i)(1-i) = 2$



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