

## STEP II, 2021, Q1 MS

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$$\begin{aligned}\cos(3a + a) &\equiv \cos 3a \cos a - \sin 3a \sin a && \mathbf{M1} \\ \cos(3a - a) &\equiv \cos 3a \cos a + \sin 3a \sin a \\ \cos 4a + \cos 2a &\equiv 2 \cos 3a \cos a\end{aligned}$$

$$\cos a \cos 3a \equiv \frac{1}{2}(\cos 4a + \cos 2a) \quad \mathbf{AG} \quad \mathbf{A1}$$

$$\begin{aligned}\sin(3a + a) &\equiv \sin 3a \cos a + \cos 3a \sin a \\ \sin(3a - a) &\equiv \sin 3a \cos a - \cos 3a \sin a \\ \sin 4a - \sin 2a &\equiv 2 \cos 3a \sin a\end{aligned}$$

$$\sin a \cos 3a \equiv \frac{1}{2}(\sin 4a - \sin 2a) \quad \mathbf{B1}$$

(i)

$$\begin{aligned}2 \cos 2x (2 \cos x \cos 3x) &= 1 && \mathbf{M1} \\ 2 \cos 2x (\cos 4x + \cos 2x) &= 1 && \mathbf{M1} \\ 2 \cos 2x (2 \cos^2 2x + \cos 2x - 1) &= 1 && \mathbf{M1} \\ 4 \cos^3 2x + 2 \cos^2 2x - 2 \cos 2x - 1 &= 0 \\ (2 \cos^2 2x - 1)(2 \cos 2x + 1) &= 0 && \mathbf{M1} \\ &&& \mathbf{A1}\end{aligned}$$

Either  $\cos^2 2x = \frac{1}{2}$ :

$$\begin{aligned}2x &= \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \\ x &= \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}\end{aligned} \quad \mathbf{A1}$$

Or  $\cos 2x = -\frac{1}{2}$ :

$$\begin{aligned}2x &= \frac{2\pi}{3}, \frac{4\pi}{3} \\ x &= \frac{\pi}{3}, \frac{2\pi}{3}\end{aligned} \quad \mathbf{A1}$$

Therefore:

$$x = \frac{\pi}{8}, \frac{\pi}{3}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{2\pi}{3}, \frac{7\pi}{8}$$



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(ii)  $2 \cos x \sin 3x \equiv \sin 4x + \sin 2x$  **B1**

$$\tan x = \tan 2x \tan 3x \tan 4x$$
 **M1**

$$\sin x \cos 2x \cos 3x \cos 4x = \cos x \sin 2x \sin 3x \sin 4x$$

$$(2 \sin x \cos 3x) \cos 2x \cos 4x = (2 \cos x \sin 3x) \sin 2x \sin 4x$$
 **M1**

$$(\sin 4x - \sin 2x) \cos 2x \cos 4x = (\sin 4x + \sin 2x) \sin 2x \sin 4x$$

$$\sin 4x (\cos 2x \cos 4x - \sin 2x \sin 4x) = \sin 2x (\cos 2x \cos 4x + \sin 2x \sin 4x)$$

$$\sin 4x \cos 6x = \sin 2x \cos 2x$$
 **M1**

$$\sin 4x \cos 6x = \frac{1}{2} \sin 4x$$
 **M1**

$$\sin 4x (2 \cos 6x - 1) = 0$$
 **A1**

Therefore  $\cos 6x = \frac{1}{2}$  or  $\sin 4x = 0$ . **AG**

$$\cos 6x = \frac{1}{2}:$$

$$6x = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}, \frac{13\pi}{3}, \frac{17\pi}{3}$$

$$x = \frac{\pi}{18}, \frac{5\pi}{18}, \frac{7\pi}{18}, \frac{11\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}$$
 **A1**

$$\sin 4x = 0:$$

$$4x = 0, \pi, 2\pi, 3\pi, 4\pi$$

$$x = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi$$
 **A1**

$\tan x$  is undefined at  $x = \frac{\pi}{2}$  **B1**

$\tan 2x$  is undefined at  $x = \frac{\pi}{4}, \frac{3\pi}{4}$  **B1**

So these are not solutions of the equation. **B1**

$$x = 0, \frac{\pi}{18}, \frac{5\pi}{18}, \frac{7\pi}{18}, \frac{11\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}, \pi$$



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