STEP II, 2021, Q10 MS

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(i) To remain stationary relative to the train the bead would have to have horizontal acceleration a. **E1**

There is no horizontal force on the bead at the origin, so this is impossible. **E1**

(ii) When the particle is at the point (x, y):

Let the angle that the tangent to the curve makes with the horizontal be θ:

The wire is smooth, so gravity will be the only force with a component in the direction of the tangent to the curve.

The acceleration of the particle will be $\begin{pmatrix} \ddot{x} - a \\ \ddot{y} \end{pmatrix}$ **B1**

Therefore, resolving in the tangential direction: **M1**

$$m(\ddot{x} - a)cos\theta + m\ddot{y}sin\theta = -mgsin\theta$$ **A1**

A1

$$(\ddot{x} - a) + (\ddot{y} + g)tan\theta = 0$$

$$\dot{y} = \dot{x}tan\theta$$ **M1**

Therefore

$$\dot{x}(\ddot{x} - a) + (\ddot{y} + g)\dot{x}tan\theta = 0$$ **M1**

$$\dot{x}(\ddot{x} - a) + (\ddot{y} + g)\dot{y} = 0$$ **A1**

$$\frac{d}{dt}\left(\frac{1}{2}(\dot{x}^2 + \dot{y}^2) - ax + gy\right) = \dot{x}(\ddot{x} - a) + (\ddot{y} + g)\dot{y} = 0$$ **M1**

So the expression is constant during the motion. **A1**

(iii) Initially, $\frac{1}{2}(\dot{x}^2 + \dot{y}^2) - ax + gy = 0$ (and throughout the motion since it is constant) **M1**

At the maximum vertical displacement $\dot{y} = 0$.

$\dot{x} = 0$ as well would only be possible at the origin (which is not maximum vertical displacement, therefore $\dot{x} = 0$ and $x \neq 0$ **M1**

Therefore, $ax = gy$

and so $g^2y^2 = a^2x^2 = a^2ky$ **M1**

Therefore, b satisfies

$$g^2b^2 = a^2kb$$

$$b = \frac{a^2k}{g^2}$$ **A1**

(iv) The square of the speed relative to the train is

$\dot{x}^2 + \dot{y}^2 = 2(ax + gy)$ **M1**

$$2\left(ax - \frac{gx^2}{k}\right)$$

M1

$$-\frac{2g}{k}\left(x - \frac{ak}{2g}\right)^2 + \frac{a^2k}{2g}$$

A1

Maximum speed is $a\sqrt{\frac{k}{2g}}$ **A1**

When $x = \frac{ak}{2g}$ **A1**