

STEP II, 2021, Q10

- 10 A train moves westwards on a straight horizontal track with constant acceleration a , where $a > 0$. Axes are chosen as follows: the origin is fixed in the train; the x -axis is in the direction of the track with the positive x -axis pointing to the East; and the positive y -axis points vertically upwards.

A smooth wire is fixed in the train. It lies in the x - y plane and is bent in the shape given by $ky = x^2$, where k is a positive constant. A small bead is threaded onto the wire. Initially, the bead is held at the origin. It is then released.

- (i) Explain why the bead cannot remain stationary relative to the train at the origin.
- (ii) Show that, in the subsequent motion, the coordinates (x, y) of the bead satisfy

$$\dot{x}(\ddot{x} - a) + \dot{y}(\ddot{y} + g) = 0$$

and deduce that $\frac{1}{2}(\dot{x}^2 + \dot{y}^2) - ax + gy$ is constant during the motion.

- (iii) Find an expression for the maximum vertical displacement, b , of the bead from its initial position in terms of a , k and g .
- (iv) Find the value of x for which the speed of the bead relative to the train is greatest and give this maximum speed in terms of a , k and g .



NextStepMaths.com

To view mark schemes, fully worked solutions and examiner's comments, and for more details about tutoring and other services offered, go to [NextStepMaths.com](https://www.NextStepMaths.com)