STEP II, 2020, Q9 MS

9	If the particles collide at time t: $Vt + Ut\cos\theta = d$, and $h - \frac{1}{2}gt^2 = Ut\sin\theta - \frac{1}{2}gt^2$ (or $h = Ut\sin\theta$)
	Therefore, $d\sin\theta - h\cos\theta = Vt\sin\theta + Ut\sin\theta\cos\theta - Ut\sin\theta\cos\theta$ $= \frac{Vh}{U}$
9(i)	Dividing the previous result by $d\cos\theta$ gives: $\tan\theta - \frac{h}{d} = \frac{Vh}{Ud\cos\theta} > 0$
	Since $\tan\beta = \frac{h}{d}$, $\tan\theta > \tan\beta$ and so $\theta > \beta$
9(ii)	The height of collision must be non-negative, so $Ut\sin\theta - \frac{1}{2}gt^2 \geq 0$.
	So $U\sin\theta \geq \frac{1}{2}gt = \frac{1}{2}g\frac{h}{U\sin\theta}$ or $(U\sin\theta)^2 \geq \frac{gh}{2}$ Therefore $U\sin\theta \geq \sqrt{\frac{gh}{2}}$.
9(iii)	$d\sin\theta - h\cos\theta$ can be written as $\sqrt{d^2+h^2}\sin(\theta-\beta)$
	So $d\sin\theta - h\cos\theta < \sqrt{d^2+h^2}$ (since $\theta > \beta$)
	Therefore, $\frac{Vh}{U} < \sqrt{d^2+h^2}$ or $\sin\beta = \frac{h}{\sqrt{d^2+h^2}} < \frac{U}{V}$
	The height at which the particles collide is: $h - \frac{1}{2}gt^2 = h - \frac{gh^2}{2U^2\sin^2\theta}$
	$h - \frac{gh^2}{2U^2\sin^2\theta} > \frac{1}{2}h$ iff $U^2\sin^2\theta > gh$
	The vertical velocity of the particle fired from B at the point of collision is: $U\sin\theta - gt = U\sin\theta - \frac{gh}{U\sin\theta}$
	$U\sin\theta - \frac{gh}{U\sin\theta} > 0$ iff $U^2\sin^2\theta > gh$
	Since both cases have the same condition: The particles collide at a height greater than $\frac{1}{2}h$ if and only if the particle projected from B is moving upwards at the time of collision.

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