

STEP II, 2020, Q8

- 8 In this question, $f(x)$ is a quartic polynomial where the coefficient of x^4 is equal to 1, and which has four real roots, 0, a , b and c , where $0 < a < b < c$.

$F(x)$ is defined by $F(x) = \int_0^x f(t) dt$.

The area enclosed by the curve $y = f(x)$ and the x -axis between 0 and a is equal to that between b and c , and half that between a and b .

- (i) Sketch the curve $y = F(x)$, showing the x co-ordinates of its turning points.

Explain why $F(x)$ must have the form $F(x) = \frac{1}{5}x^2(x - c)^2(x - h)$, where $0 < h < c$.

Find, in factorised form, an expression for $F(x) + F(c - x)$ in terms of c , h and x .

- (ii) If $0 \leq x \leq c$, explain why $F(b) + F(x) \geq 0$ and why $F(b) + F(x) > 0$ if $x \neq a$.
Hence show that $c - b = a$ or $c > 2h$.

By considering also $F(a) + F(x)$, show that $c = a + b$ and that $c = 2h$.

- (iii) Find an expression for $f(x)$ in terms of c and x only.

Show that the points of inflection on $y = f(x)$ lie on the x -axis.



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