STEP II, 2020, Q7 MS

7(i)	$\lvert w-1\rvert^2 = \left\lvert\frac{1-ti}{1+ti}\right\rvert^2 = \frac{(1-ti)(1+ti)}{(1+ti)(1-ti)} = 1$, which is independent of t.
	Points on the line $Re(z) = 3$ have the form $z = 3 + ti$ and the points satisfying $\lvert w-1\rvert = 1$ lie on a circle with centre 1.
	If $z = p + ti$, then $\lvert w-c\rvert^2 = \left\lvert\frac{2-(p-2)c-cti}{(p-2)+ti}\right\rvert^2 = \frac{(2-(p-2)c)^2+c^2t^2}{(p-2)^2+t^2}$
	which is independent of t when $(2-(p-2)c)^2 = c^2(p-2)^2$
	which is when $c = \frac{1}{p-2}$. Thus the circle has centre at $\frac{1}{p-2}$ and radius $\frac{1}{\lvert p-2\rvert}$.
	$w = \frac{2}{(p-2)+ti} = \frac{2(p-2)-2ti}{(p-2)^2+t^2}$,
	so $\text{Im}(w) > 0$ when $t < 0$; that is, for those z on V with negative imaginary part.
7(ii)	If $z = t + qi$ then $\lvert w-ci\rvert^2 = \left\lvert\frac{2+cq-(t-2)ci}{(t-2)+qi}\right\rvert^2 = \frac{c^2(t-2)^2+(cq+2)^2}{(t-2)^2+q^2}$
	which is independent of t when $(cq+2)^2 = c^2q^2$
	which is when $c = -\frac{1}{q}$ so the circle has centre $-\frac{1}{q}i$ **A1** and radius $\sqrt{c^2} = \frac{1}{\lvert q\rvert}$ **A1**.
	$w = \frac{2}{(t-2)+qi} = \frac{2(t-2)-2qi}{(t-2)^2+q^2}$,
	so $\text{Re}(w) > 0$ when $t > 2$; that is, for those z on H with real part greater than 2.