

## STEP II, 2020, Q7 MS

7(i)	$ w-1 ^2 = \left  \frac{1-ti}{1+ti} \right ^2 = \frac{(1-ti)(1+ti)}{(1+ti)(1-ti)} = 1$ , which is independent of $t$ .
	Points on the line $Re(z) = 3$ have the form $z = 3 + ti$ and the points satisfying $ w-1  = 1$ lie on a circle with centre 1.
	If $z = p + ti$ , then $ w-c ^2 = \left  \frac{2-(p-2)c-cti}{(p-2)+ti} \right ^2 = \frac{(2-(p-2)c)^2 + c^2t^2}{(p-2)^2 + t^2}$
	which is independent of $t$ when $(2-(p-2)c)^2 = c^2(p-2)^2$
	which is when $c = \frac{1}{p-2}$ . Thus the circle has centre at $\frac{1}{p-2}$ and radius $\frac{1}{ p-2 }$ .
	$w = \frac{2}{(p-2)+ti} = \frac{2(p-2)-2ti}{(p-2)^2+t^2},$
	so $Im(w) > 0$ when $t < 0$ ; that is, for those $z$ on $V$ with negative imaginary part.
7(ii)	If $z = t + qi$ then $ w-ci ^2 = \left  \frac{2+cq-(t-2)ci}{(t-2)+qi} \right ^2 = \frac{c^2(t-2)^2 + (cq+2)^2}{(t-2)^2 + q^2}$
	which is independent of $t$ when $(cq+2)^2 = c^2q^2$
	which is when $c = -\frac{1}{q}$ so the circle has centre $-\frac{1}{q}i$ <b>A1</b> and radius $\sqrt{c^2} = \frac{1}{ q }$ <b>A1</b> .
	$w = \frac{2}{(t-2)+qi} = \frac{2(t-2)-2qi}{(t-2)^2+q^2},$
	so $Re(w) > 0$ when $t > 2$ ; that is, for those $z$ on $H$ with real part greater than 2.



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