STEP II, 2020, Q5 EC

This was a popular question and many of the solutions made good progress on the early parts of the question.

The majority of candidates gained full marks for part (i), but some candidates did not mention that $x - d(x) \geq 0$.

There was a wide range of marks achieved on part (ii). The proof that $x - 44d(x)$ is a multiple of 9 if and only if x is a multiple of 9 was completed well by those who managed to prove the result, but the majority of other attempts seen did not score any marks. In a small number of cases only the "if" direction was proved. Those who were unable to prove the first result in this part were often able to continue and find the required bounds on x however. Candidates who had completed both of these parts generally managed to find the correct answer $x = 792$, but did not necessarily fully justify that it was the only one.

Most candidates scored low marks on part (iii). It was very common to see an insufficient proof that $9|x$. Without guidance from the question as to how to find bounds on x, students produced a wide range of approaches; better bounds were needed if the student only used $107|x$, but the simple bound $d\big(d(x)\big) \leq d(x)$ together with divisibility by 963 was sufficient.