STEP II, 2020, Q5

5 If x is a positive integer, the value of the function $\mathrm{d}(x)$ is the sum of the digits of x in base 10. For example, $\mathrm{d}(249) = 2 + 4 + 9 = 15$.

An n-digit positive integer x is written in the form $\sum_{r=0}^{n-1} a_r \times 10^r$, where $0 \leqslant a_r \leqslant 9$ for all $0 \leqslant r \leqslant n-1$ and $a_{n-1} > 0$.

(i) Prove that $x - \mathrm{d}(x)$ is non-negative and divisible by 9.

(ii) Prove that $x - 44\mathrm{d}(x)$ is a multiple of 9 if and only if x is a multiple of 9.

Suppose that $x = 44\mathrm{d}(x)$. Show that if x has n digits, then $x \leqslant 396n$ and $x \geqslant 10^{n-1}$, and hence that $n \leqslant 4$.

Find a value of x for which $x = 44\mathrm{d}(x)$. Show that there are no further values of x satisfying this equation.

(iii) Find a value of x for which $x = 107\mathrm{d}\left(\mathrm{d}(x)\right)$. Show that there are no further values of x satisfying this equation.

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