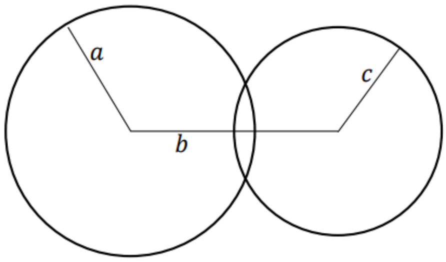
STEP II, 2020, Q4 MS

4(i)	The straight line distance between two points must be less than the length of any other rectilinear path between the points.
4(ii)	
	Diagram showing two circles and straight line joining their centres. Length of line and radii of circles are a, b and c in some order.
	Either statement that the straight line is the longest of the lengths, or explanation that one circle cannot be contained inside the other.
	Explanation that the circles must meet.
4(iii)	**(A)** If $a+b>c$ then $(a+1)+(b+1)>c+2>c+1$ *et cycl.*, so $a+1, b+1, c+1$ can always form the sides of a triangle.
	(B) If $a=b=c=1$ we have $1, 1, 1$ which can form the sides of a triangle.
	If $a=1, b=c=2$ we have $\frac{1}{2}$, $1, 2$ which cannot form the sides of a triangle.
	Therefore, $\frac{a}{b}, \frac{b}{c}, \frac{c}{a}$ can sometimes, but not always form the sides of a triangle.
	(C) If $p\geq q\geq r$ then $\|p-q\|+\|q-r\|=p-q+q-r=p-q=\|p-r\|$
	So two of $\|p-q\|$, $\|q-r\|$, $\|p-r\|$ will always sum to the third, so they never form the sides of a triangle.
	(D) If $a+b>c$ then $a^2+bc+b^2+ca=a^2+b^2-2ab+c(a+b)+2ab$
	$=(a-b)^2+c(a+b)+2ab>c^2+ab$ *et cycl.* so a^2+bc, b^2+ca, c^2+ab can always form the sides of a triangle.
4(iv)	Since $a+b>a$ and b, $\frac{f(a)}{a}>\frac{f(a+b)}{a+b}$ and $\frac{f(b)}{b}>\frac{f(a+b)}{a+b}$
	Since $c<a+b$, $f(c)<f(a+b)$
	Thus $f(a)+f(b)>\frac{af(a)}{a+b}+\frac{bf(b)}{a+b}=f(a+b)>f(c)$ *et cycl.* So $f(a)$, $f(b)$ and $f(c)$ can form the sides of a triangle.