STEP II, 2020, Q4 EC

Most candidates could justify the triangle inequality in part (i) (as well as arguing that the shortest distance is the straight line, there were successful uses of the cosine rule or $c = a\cos B + b\cos A$), but were less confident in proving the converse in part (ii). Successful approaches were to consider two circular loci for the SSS construction, or to fix two sides and vary the angle between them; in both cases care was needed to ensure all three inequalities were actually used, for example checking that neither circle can contain the other, which was often omitted. There was one elegant solution using three pairwise tangent circles.

A reasonable number of candidates obtained correct answers of “always”, “sometimes” (by examples) and “never” for part (iii) A, B and C respectively, although it was surprisingly common to forget that a, b, c must be the sides of a triangle. However, B caused some confusion as many candidates spent some time trying to prove that the new lengths did satisfy the triangle inequality.

Parts (iii) D and (iv) were found much harder and many candidates did not attempt them. A reasonable number of candidates were able to make some progress, but there were few complete solutions to either of these parts. Common errors for (iii) D were showing that the sum of the three inequalities is a true statement, or attempting to prove a positive result by examples. Most substantial attempts at (iv) used a different approach of fixing the order of a, b, c and reducing the problem to proving one of the three inequalities.