

STEP II, 2020, Q4

- 4 (i) Given that a , b and c are the lengths of the sides of a triangle, explain why $c < a + b$, $a < b + c$ and $b < a + c$.
- (ii) Use a diagram to show that the converse of the result in part (i) also holds: if a , b and c are positive numbers such that $c < a + b$, $a < b + c$ and $b < c + a$ then it is possible to construct a triangle with sides of length a , b and c .
- (iii) When a , b and c are the lengths of the sides of a triangle, determine in each case whether the following sets of three lengths can
- always
 - sometimes but not always
 - never

form the sides of a triangle. Prove your claims.

(A) $a + 1, b + 1, c + 1$.

(B) $\frac{a}{b}, \frac{b}{c}, \frac{c}{a}$.

(C) $|a - b|, |b - c|, |c - a|$.

(D) $a^2 + bc, b^2 + ca, c^2 + ab$.

- (iv) Let f be a function defined on the positive real numbers and such that, whenever $x > y > 0$,

$$f(x) > f(y) > 0 \text{ but } \frac{f(x)}{x} < \frac{f(y)}{y}.$$

Show that, whenever a , b and c are the lengths of the sides of a triangle, then $f(a)$, $f(b)$ and $f(c)$ can also be the lengths of the sides of a triangle.



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