

## STEP II, 2020, Q2

- 2 The curves  $C_1$  and  $C_2$  both satisfy the differential equation

$$\frac{dy}{dx} = \frac{kxy - y}{x - kxy},$$

where  $k = \ln 2$ .

All points on  $C_1$  have positive  $x$  and  $y$  co-ordinates and  $C_1$  passes through  $(1, 1)$ . All points on  $C_2$  have negative  $x$  and  $y$  co-ordinates and  $C_2$  passes through  $(-1, -1)$ .

- (i) Show that the equation of  $C_1$  can be written as  $(x - y)^2 = (x + y)^2 - 2^{x+y}$ .

Determine a similar result for curve  $C_2$ .

Hence show that  $y = x$  is a line of symmetry of each curve.

- (ii) Sketch on the same axes the curves  $y = x^2$  and  $y = 2^x$ , for  $x \geq 0$ . Hence show that  $C_1$  lies between the lines  $x + y = 2$  and  $x + y = 4$ .

Sketch curve  $C_1$ .

- (iii) Sketch curve  $C_2$ .



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