

STEP II, 2020, Q12

12 The score shown on a biased n -sided die is represented by the random variable X which has distribution $P(X = i) = \frac{1}{n} + \varepsilon_i$ for $i = 1, 2, \dots, n$, where not all the ε_i are equal to 0.

(i) Find the probability that, when the die is rolled twice, the same score is shown on both rolls. Hence determine whether it is more likely for a fair die or a biased die to show the same score on two successive rolls.

(ii) Use part (i) to prove that, for any set of n positive numbers x_i ($i = 1, 2, \dots, n$),

$$\sum_{i=2}^n \sum_{j=1}^{i-1} x_i x_j \leq \frac{n-1}{2n} \left(\sum_{i=1}^n x_i \right)^2.$$

(iii) Determine, with justification, whether it is more likely for a fair die or a biased die to show the same score on three successive rolls.



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