

STEP II, 2020, Q11

11 A coin is tossed repeatedly. The probability that a head appears is p and the probability that a tail appears is $q = 1 - p$.

- (i) A and B play a game. The game ends if two successive heads appear, in which case A wins, or if two successive tails appear, in which case B wins.

Show that the probability that the game never ends is 0.

Given that the first toss is a head, show that the probability that A wins is $\frac{p}{1 - pq}$.

Find and simplify an expression for the probability that A wins.

- (ii) A and B play another game. The game ends if three successive heads appear, in which case A wins, or if three successive tails appear, in which case B wins.

Show that

$$P(\text{A wins} \mid \text{the first toss is a head}) = p^2 + (q + pq) P(\text{A wins} \mid \text{the first toss is a tail})$$

and give a similar result for $P(\text{A wins} \mid \text{the first toss is a tail})$.

Show that

$$P(\text{A wins}) = \frac{p^2(1 - q^3)}{1 - (1 - p^2)(1 - q^2)}.$$

- (iii) A and B play a third game. The game ends if a successive heads appear, in which case A wins, or if b successive tails appear, in which case B wins, where a and b are integers greater than 1.

Find the probability that A wins this game.

Verify that your result agrees with part (i) when $a = b = 2$.



NextStepMaths.com

To view mark schemes, fully worked solutions and examiner's comments, and for more details about tutoring and other services offered, go to NextStepMaths.com