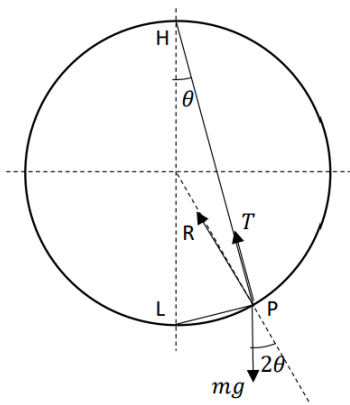


STEP II, 2020, Q10 MS

10(i)	
	Diagram showing necessary forces and angles
	$T = \frac{\lambda(2a \cos \alpha - l)}{l}$
	Resolving tangentially: $T \sin \alpha - mg \sin 2\alpha = 0$
	Therefore $\sin \alpha \left(\frac{\lambda}{l}(2a \cos \alpha - l) - 2mg \cos \alpha \right) = 0$
	Since $\sin \alpha > 0$, $2a\lambda \cos \alpha - \lambda l - 2mgl \cos \alpha = 0$ $\cos \alpha = \frac{\lambda l}{2(a\lambda - mgl)}$
	$\cos \alpha < 1$, so $\lambda l < 2(a\lambda - mgl)$ Therefore $\lambda(2a - l) > 2mgl$
	Since $2a - l > 0$, $\lambda > \frac{2mgl}{2a - l}$
10(ii)	Energy: $\frac{1}{2}mv^2 - mga \cos 2\theta + \frac{\lambda}{2l}(2a \cos \theta - l)^2 = \frac{1}{2}mu^2 - mga + \frac{\lambda}{2l}(2a - l)^2$
	If the particle comes to rest when $\theta = \beta$: $-mga(2\cos^2 \beta - 1) + \frac{\lambda}{2l}(2a \cos \beta - l)^2 = \frac{1}{2}mu^2 - mga + \frac{\lambda}{2l}(2a - l)^2$
	$a\lambda \cos^2 \beta \left(\frac{2(a\lambda - mgl)}{\lambda l} \right) - 2a\lambda \cos \beta = \frac{1}{2}mu^2 - 2mga + \frac{2\lambda a^2}{l} - 2a\lambda$
	Therefore, $\cos^2 \beta - 2 \cos \alpha \cos \beta = \frac{mu^2}{2a\lambda} \cos \alpha + 1 - 2 \cos \alpha$
	Adding $\cos^2 \alpha$ to both sides: $(\cos \alpha - \cos \beta)^2 = (1 - \cos \alpha)^2 + \frac{mu^2}{2a\lambda} \cos \alpha$
	For this to occur, $\cos \beta > 0$:
	$\cos^2 \alpha > (1 - \cos \alpha)^2 + \frac{mu^2}{2a\lambda} \cos \alpha$
	And so, $u^2 < \frac{2a\lambda}{m}(2 - \sec \alpha)$



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