

## STEP II, 2020, Q10

- 10** A particle  $P$  of mass  $m$  moves freely and without friction on a wire circle of radius  $a$ , whose axis is horizontal. The highest point of the circle is  $H$ , the lowest point of the circle is  $L$  and angle  $PHL = \theta$ . A light spring of modulus of elasticity  $\lambda$  is attached to  $P$  and to  $H$ . The natural length of the spring is  $l$ , which is less than the diameter of the circle.

- (i) Show that, if there is an equilibrium position of the particle at  $\theta = \alpha$ , where  $\alpha > 0$ , then  $\cos \alpha = \frac{\lambda l}{2(a\lambda - mgl)}$ .

Show also that there will only be such an equilibrium position if  $\lambda > \frac{2mgl}{2a - l}$ .

When the particle is at the lowest point  $L$  of the circular wire, it has speed  $u$ .

- (ii) Show that, if the particle comes to rest before reaching  $H$ , it does so when  $\theta = \beta$ , where  $\cos \beta$  satisfies

$$(\cos \alpha - \cos \beta)^2 = (1 - \cos \alpha)^2 + \frac{mu^2}{2a\lambda} \cos \alpha,$$

where  $\cos \alpha = \frac{\lambda l}{2(a\lambda - mgl)}$ .

Show also that this will only occur if  $u^2 < \frac{2a\lambda}{m}(2 - \sec \alpha)$ .



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