STEP II, 2019, Q9 MS

In part (i), the position vector of the particle at time t can be calculated. The distance OP is then the modulus of this vector. It is easier to differentiate the square of the distance with respect to time (which is sufficient as this will be increasing if and only if the distance is increasing). The resulting expression can be shown to be positive if $\sin\alpha < \frac{2\sqrt{2}}{3}$. Similarly, in the case where $\sin\alpha > \frac{2\sqrt{2}}{3}$ it is possible to identify a value of t for which the distance is certainly decreasing and show that this is before the moment at which the particle lands.

In part (ii), the vector QP can again be calculated and then the distance PQ found by taking the modulus. As in part (i) it is simpler to deal with PQ^2 rather than PQ. In this case, care must be taken with the inequality to check that both sides are positive before they are squared and used to justify that the distance is increasing throughout the flight of P.

9	(i)		$\boldsymbol{r} = \begin{pmatrix} ut\cos\alpha \\ ut\sin\alpha - \frac{1}{2}gt^2 \end{pmatrix}$ $\boldsymbol{r}^2 = u^2t^2\cos^2\alpha + u^2t^2\sin^2\alpha - ugt^3\sin\alpha + \frac{1}{4}g^2t^4$ $= u^2t^2 - ugt^3\sin\alpha + \frac{1}{4}g^2t^4$ $\frac{d}{dt}(\boldsymbol{r}^2) = 2u^2t - 3ugt^2\sin\alpha + g^2t^3$ $= t(2u^2 - 3ugt\sin\alpha + g^2t^2)$ $= t(2u^2 - \frac{9}{4}u^2\sin^2\alpha + (gt - \frac{3}{2}u\sin\alpha)^2)$ If $\sin\alpha < \frac{2\sqrt{2}}{3}$, then $2u^2 - \frac{9}{4}u^2\sin^2\alpha > 0$ and distance is always increasing. If $\sin\alpha > \frac{2\sqrt{2}}{3}$, then distance is decreasing at $t = \frac{3u}{2g}\sin\alpha$ Landing occurs at $t = \frac{2u}{g}\sin\alpha$, which is later (<u>Or</u> imagine falls through ground. Distance increasing while underground, so any decrease must be above ground)	M1 M1 A1 M1 A1 M1 M1 M1 A1 (AG) M1 E1 (11 marks)
	(ii)		$\boldsymbol{r} = \begin{pmatrix} ut\cos\alpha + vt \\ ut\sin\alpha - \frac{1}{2}gt^2 \end{pmatrix}$ $PQ^2 = (u\cos\alpha + v)^2t^2 + u^2t^2\sin^2\alpha - ugt^3\sin\alpha + \frac{1}{4}g^2t^4$ $\frac{d}{dt}(PQ^2) = 2t(u\cos\alpha + v)^2 + 2u^2t\sin^2\alpha + 2tu^2\sin^2\alpha - 3ugt^2\sin\alpha + g^2t^3$ $= t\left(2(u\cos\alpha + v)^2 + 2u^2\sin^2\alpha - \frac{9}{4}u^2\sin^2\alpha + \left(gt - \frac{3}{2}u\sin\alpha\right)^2\right)$ $= t\left(2(u\cos\alpha + v)^2 - \frac{1}{4}u^2\sin^2\alpha + \left(gt - \frac{3}{2}u\sin\alpha\right)^2\right)$ If $2\sqrt{2}v > (\sin\alpha - 2\sqrt{2}\cos\alpha)u$, then $8(u\cos\alpha + v)^2 > u^2\sin^2\alpha$ So PQ is increasing for all t.	B1 M1 A1 M1 M1 M1 A1 M1 A1 (AG) (9 marks)