STEP II, 2019, Q8 MS

In part (i), the property of f means that $f(\boldsymbol{M}) = f(\boldsymbol{MI}) = f(\boldsymbol{M})f(\boldsymbol{I})$. Note that justification of $f(\boldsymbol{I}) = 1$ requires that $f(\boldsymbol{M}) \neq 0$.

In part (ii), note that $\boldsymbol{J}^2 = \boldsymbol{I}$ and so the value of $f(\boldsymbol{J})$ must be either 1 or -1. The second result of this part follows from application of the property of function f.

For part (iii), first show that $f\left(\begin{pmatrix} a & b \\ a & b \end{pmatrix}\right) = 0$ by applying the result of part (ii) and then pre-multiply this matrix by $\boldsymbol{K}$ to obtain one in which the second row is a multiple of the first.

For part (iv), note that $\boldsymbol{P}^2 = \boldsymbol{K}^{-1}\boldsymbol{PK}$ in the case where $k = 2$. This leads to the fact that $f(\boldsymbol{P})$ must be either 0 or 1. The fact that $\boldsymbol{P}^{-1}$ exists can then be used to show that $f(\boldsymbol{P})$ cannot be 0.

8	(i)		$f(\boldsymbol{M}) = f(\boldsymbol{MI}) = f(\boldsymbol{M})f(\boldsymbol{I})$ $\Rightarrow f(\boldsymbol{I}) = 1$ since $f(\boldsymbol{M}) \neq 0$	M1 A1 (AG) (2 marks)
	(ii)		$f(J)^2 = f(J^2)$ $= f(I) = 1$ $\Rightarrow f(J) = -1$ since $f(J) \neq 1$ $f\left(\begin{pmatrix} c & d \\ a & b \end{pmatrix}\right) = f\left(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right)$ $= -f\left(\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right)$ legitimately obtained $f\left(\begin{pmatrix} d & c \\ b & a \end{pmatrix}\right) = f\left(\begin{pmatrix} c & d \\ a & b \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\right)$ $= -f\left(\begin{pmatrix} c & d \\ a & b \end{pmatrix}\right)$ legitimately obtained	M1 M1 A1 M1 A1 (AG) M1 A1 (AG) (7 marks)
	(iii)		Using first equality in previous part (or otherwise correctly justified) $f\left(\begin{pmatrix} a & b \\ a & b \end{pmatrix}\right) = -f\left(\begin{pmatrix} a & b \\ a & b \end{pmatrix}\right)$ $f\left(\begin{pmatrix} a & b \\ a & b \end{pmatrix}\right) = 0$ $f\left(\begin{pmatrix} a & b \\ ka & kb \end{pmatrix}\right) = f\left(\boldsymbol{K}\begin{pmatrix} a & b \\ a & b \end{pmatrix}\right)$ $= 0$	M1 M1 M1 A1 (AG) (4 marks)
	(iv)		$\boldsymbol{K}^{-1}\boldsymbol{PK} = \begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}$ $f(\boldsymbol{K})f(\boldsymbol{K}^{-1}) = F(\boldsymbol{I}) = 1 \Rightarrow f(\boldsymbol{K}^{-1}) = f(\boldsymbol{K})^{-1}$ $f(\boldsymbol{K}^{-1}\boldsymbol{PK}) = f(\boldsymbol{K}^{-1})f(\boldsymbol{PK})$ (must use two stages) $= f(\boldsymbol{K}^{-1})f(\boldsymbol{P})f(\boldsymbol{K})$ $= f(\boldsymbol{P})$ $\boldsymbol{P}^2 = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ $f(\boldsymbol{P}^2) = f(\boldsymbol{P}) \Rightarrow f(\boldsymbol{P}) = 0$ or 1 $\boldsymbol{P}^{-1}$ exists so $f(\boldsymbol{P})f(\boldsymbol{P}^{-1}) = 1 \Rightarrow f(\boldsymbol{P}) \neq 0$	B1 M1 M1 A1 (AG)? B1 M1 A1 (7 marks)